Facultatea de Inginerie Electrică, Energetică
şi Informatică Aplicată (IEEIA)

Identificarea şi Modelarea Sistemelor

C6

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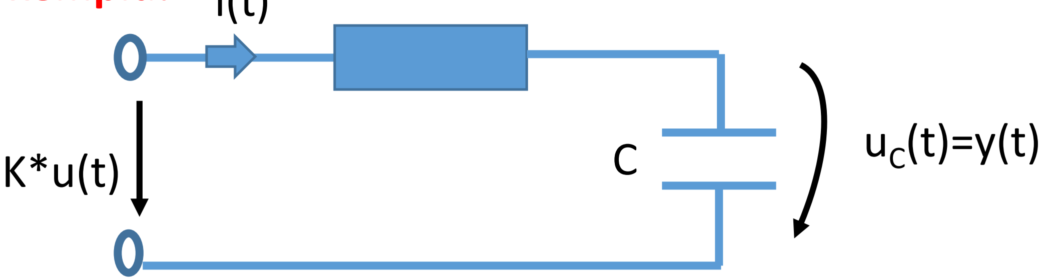
Modele grafice – raspunsul indicial Identificare bazata pe modelul unui sistem de ordin I

Exemplu:



$R*i(t)+u_C(t)=K*u(t)$

$i(t)=C*du_C(t)/dt$

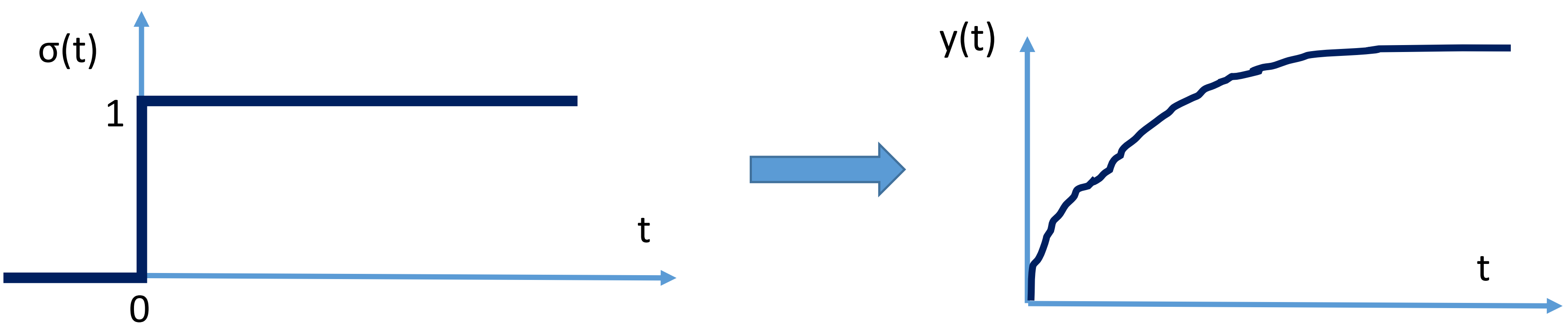
$RC*dy(t)/dt+y(t)=K*u(t)$

$$H(s) = \frac{K}{RCs+1}$$

K- amplificare;

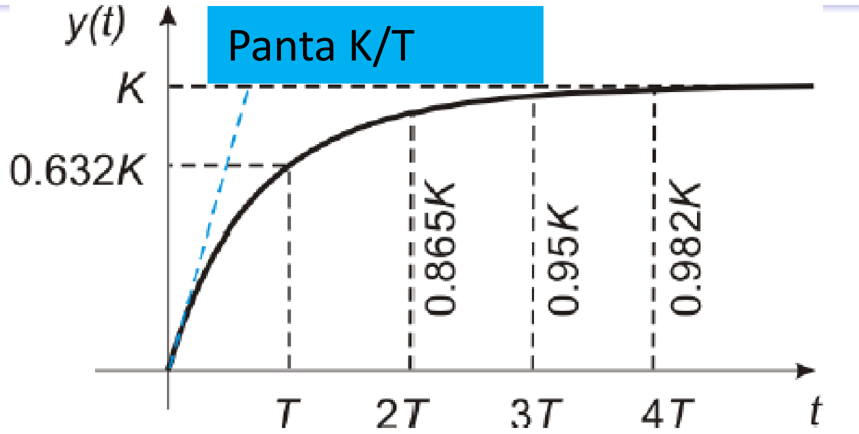
T=RC (constanta de timp)

Raspunsul indicial



Modele grafice – raspunsul indicial
Identificare bazata pe modelul unui sistem de ordin I

Raspunsul indicial – teorie TS



Prin aplicarea transformatei Laplace inverse: $$y(t) = K(1 - e^{-t/T})$$

$$\lim_{t \to \infty} y(t) = K(1 - 0) = K$$

$$\dot{y}(t) = \frac{K}{T} e^{-t/T}, \quad \dot{y}(0) = \frac{K}{T} e^{0} = \frac{K}{T}$$

$$y(T) = K(1 - e^{-1}) \approx 0.632K$$

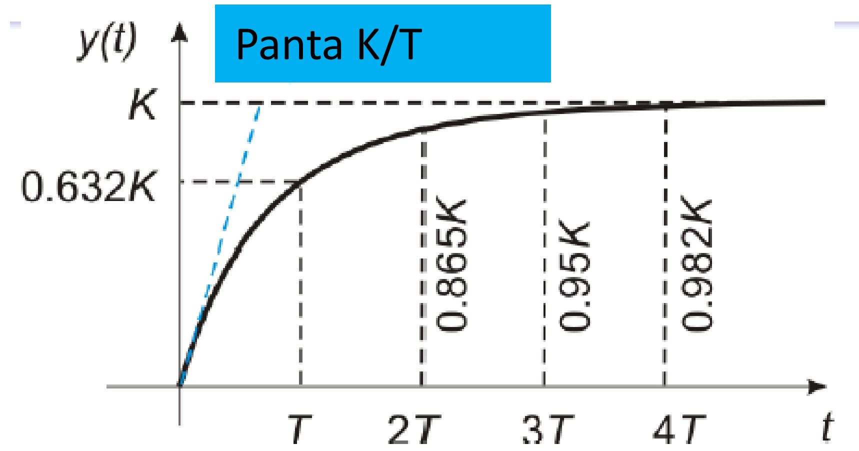
Identic, se pot calcula y(2T), y(3T), y(4T).

Modele grafice – raspunsul indicial
Identificare bazata pe modelul unui sistem de ordin I

Raspunsul indicial – aplicare algoritm pentru identificarea sistemelor

Pe baza unui raspuns indicial real (model grafic, neparametric), al unui sistem necunoscut, se va proceda la aproximarea sistemului printr-o functie de transfer (model parametric):

$$H(s) = \frac{K}{Ts+1}$$



$$y(t) = K(1 - e^{-t/T})$$

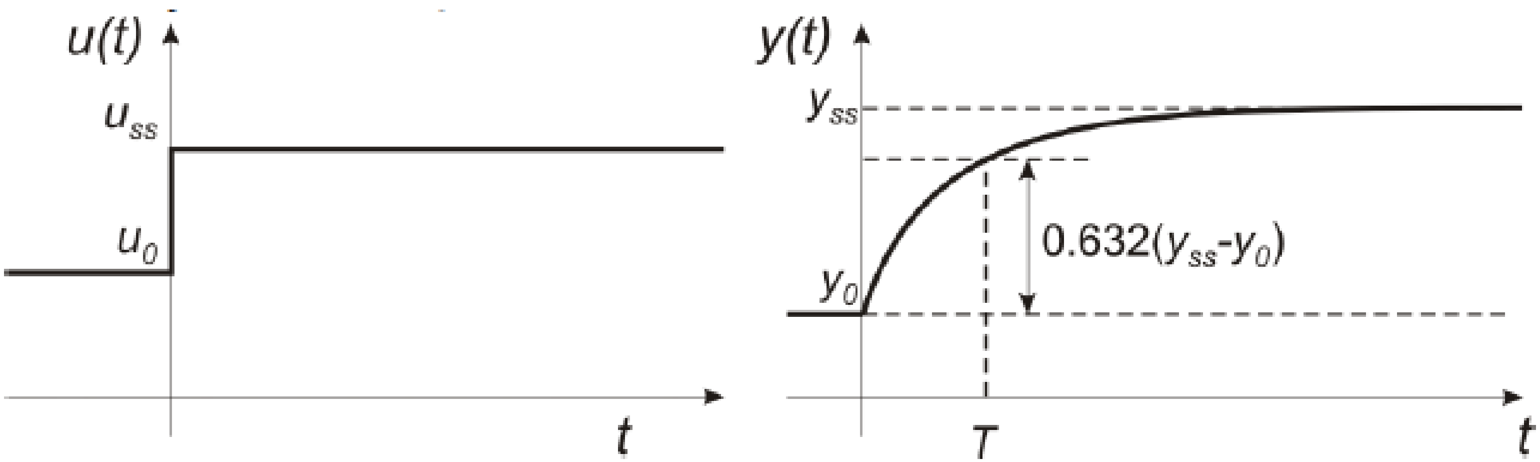
1. **Se identifica valoarea de regim stationar a lui y(t): $y_s(t)=K$;**
2. **Se determina timpul la care y(t) atinge 0.632 din valoarea $y_s(t)$. Acesta va fi egal cu T.**
3. **Se scrie functia de transfer H(s) identificata pentru sistemul real (de ordin I).**

Modele grafice – raspunsul indicial
Identificare bazata pe modelul unui sistem de ordin I

Raspunsul indicial – aplicare algoritm pentru identificarea sistemelor

Conditii initiale nenule

In practica nu pot utiliza semnale ideale tip treapta unitara (sistemul trebuie tinut in jurul unui punct de functionare);
Se presupune ca sistemul se afla intr-o stare stationara y_0 cand pe intrare avea aplicata valoarea u_0.



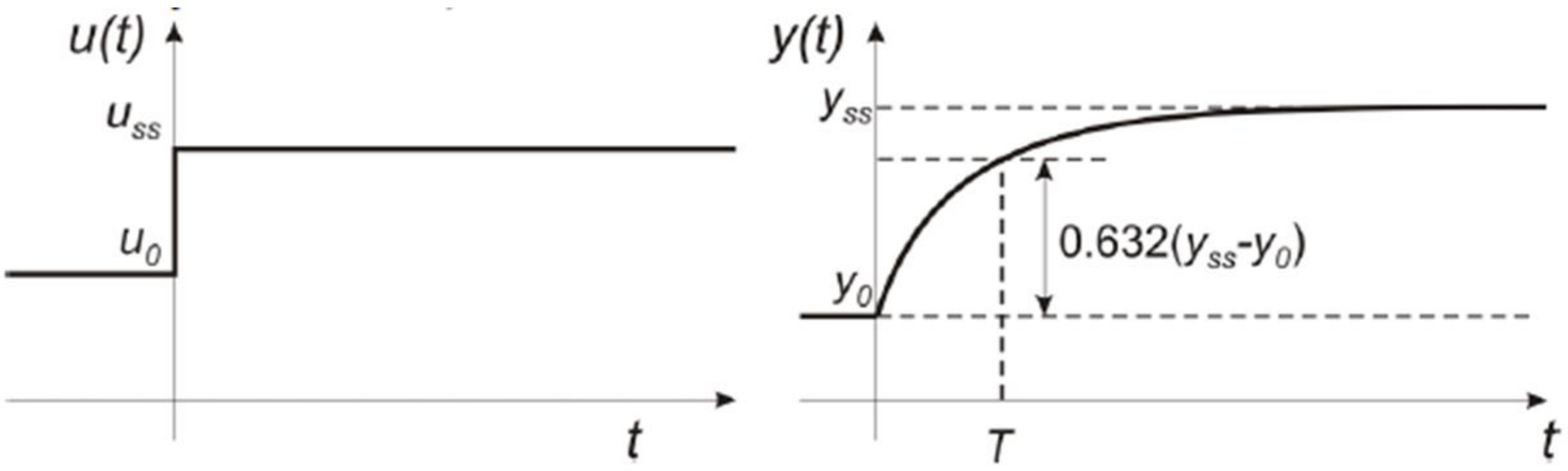
Pe baza proprietatii de liniaritate, noua intrare este (semnalul $u_s(t)$ este intrare tip treapta, ideal; iar $y_s(t)$ este iesirea raspuns la $u_s(t)$**):**

$u(t)=u_0+(u_{ss}-u_0)u_s(t)$ → $y(t)=y_0+(u_{ss}-u_0)y_s(t)$

Modele grafice – raspunsul indicial
Identificare bazata pe modelul unui sistem de ordin I

Raspunsul indicial – aplicare algoritm pentru identificarea sistemelor

Conditii initiale nenule



Rezulta:

$$y_{ss} = y_0+(u_{ss}-u_0)K$$

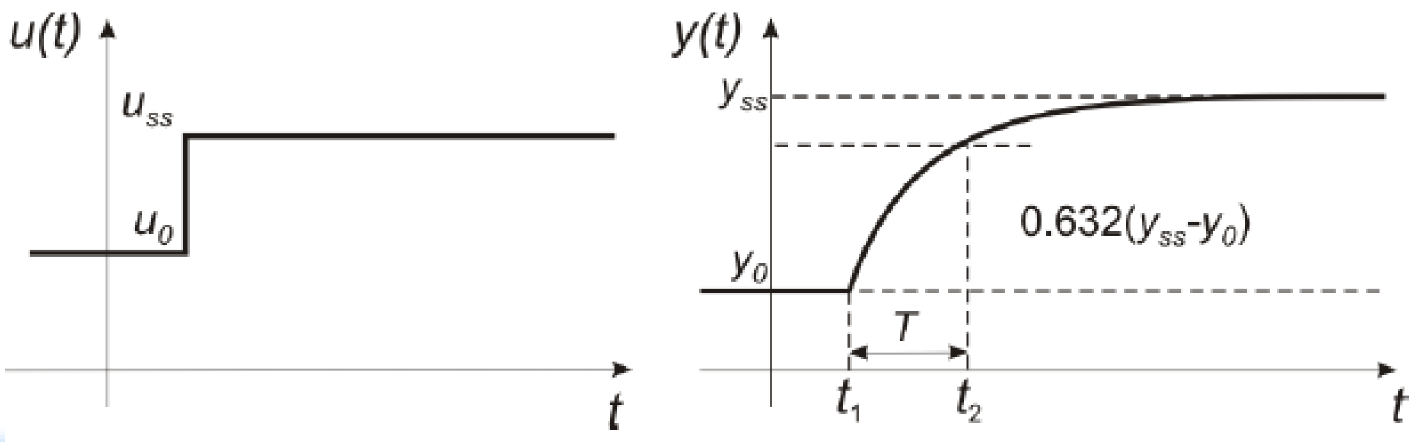
$$y(T)= y_0+ 0.632*(u_{ss}-u_0)$$

Modele grafice – raspunsul indicial
Identificare bazata pe modelul unui sistem de ordin I

Raspunsul indicial – aplicare algoritm pentru identificarea sistemelor

Conditii initiale nenule – Algoritm general

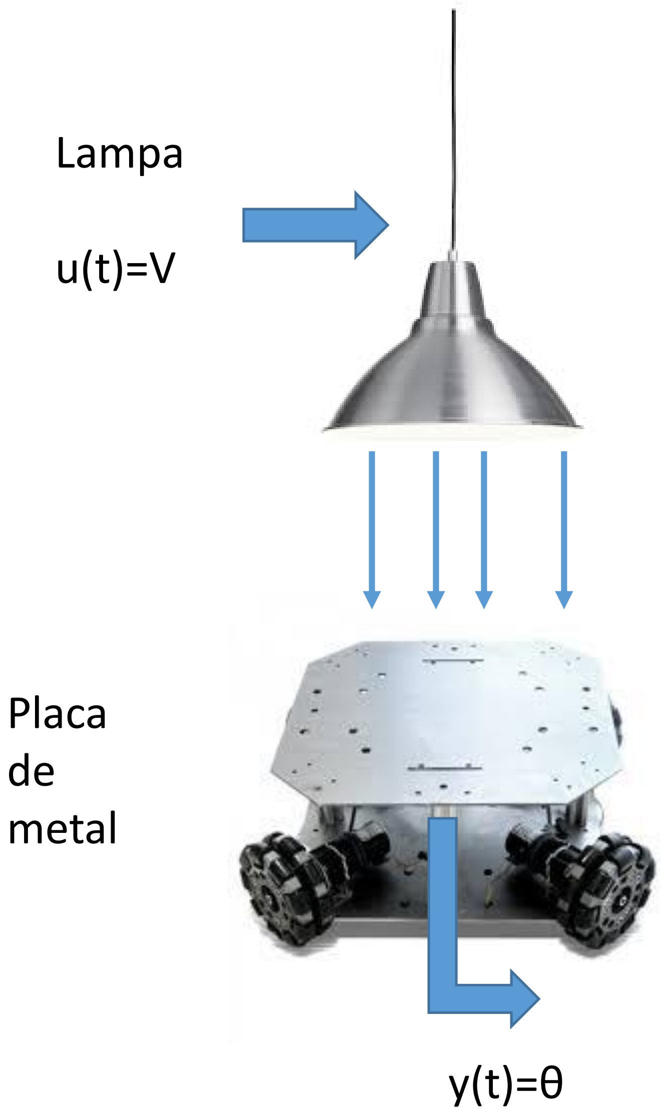
Momentul de timp la care se aplica semnalul treapta poate fi diferit de zero:



Algoritm general:

1. Se citesc valorile y_{ss} , y_0 , u_{ss} ,u_0
2. Se calculeaza $K = (y_{ss} - y_0)/(u_{ss} - u_0)$
3. Se citesc valorile t_1 (start semnal treapta) si t_2 (iesirea atinge 0.632 din valoarea ($y_{ss} - y_0$));
4. Se calculeaza $T=t_2-t_1$.

Exemplu (L. Busoniu, System Identification, UTCluj)

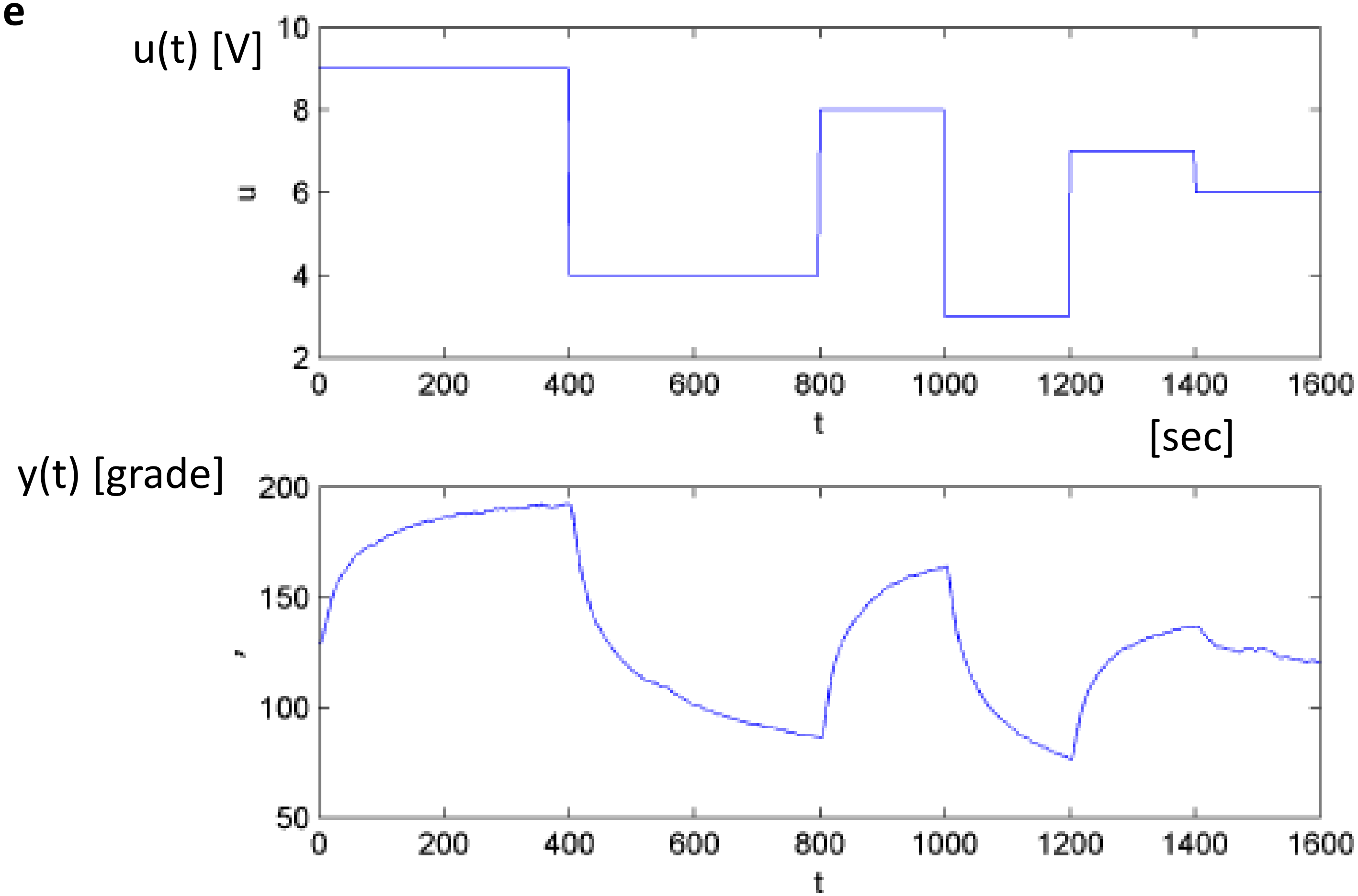


Proces de incalzire.

Intrarea este tensiunea V aplicata lampii, iar iesirea este temperatura θ citita de un termocuplu plasat pe partea de jos a placii de metal.

Exemplu (L. Busoniu, System Identification, UTCluj)

Date experimentale



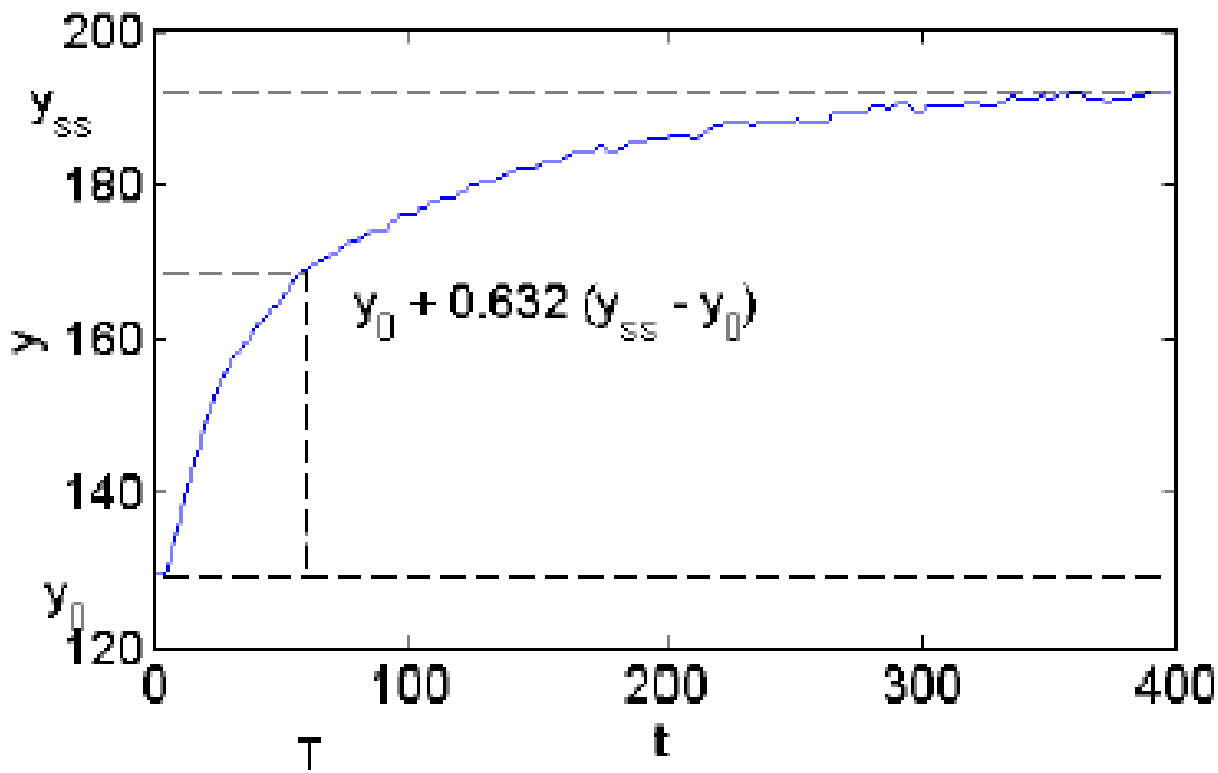
Datele sunt esantionate la T=2s. Analiza se face ca pentru un sistem continuu (datele 0-400s sunt utilizate pentru identificare si cele de la 400s-1200s pot fi utilizate pentru validare).

Exista zgomot in datele de iesire (cazul real).

Exemplu (L. Busoniu, System Identification, UTCluj)

Graficul este model neparametric. Se va estima o functie de transfer.

Y_{ss} ≈ 192°C,
y_0 ≈ 129°C.
u_{ss} = 9V
u_0=6V (din date initiale experiment)



$$K = \frac{y_{ss} - y_0}{u_{ss} - u_0} \approx \frac{192 - 129}{9 - 6} \approx 21$$

$$y(T) = y_0 + 0.632(y_{ss} - y_0) \approx 169$$

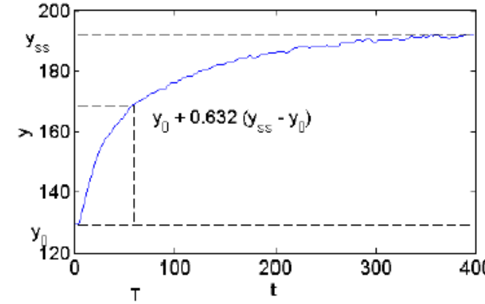
Se identifica pe axa timpului punctul pentru care **y(T)= 169°C** si rezulta **T=60s.**

Se propune functia de transfer
('^' indica valoare estimata).

$$\widehat{K} = 21$$

$$\widehat{T} = 60$$

$$\widehat{H}(s) = \frac{\widehat{K}}{\widehat{T}s + 1} = \frac{21}{60s + 1}$$



$$K = \frac{y_{ss} - y_0}{u_{ss} - u_0} \approx \frac{192 - 129}{9 - 6} \approx 21$$

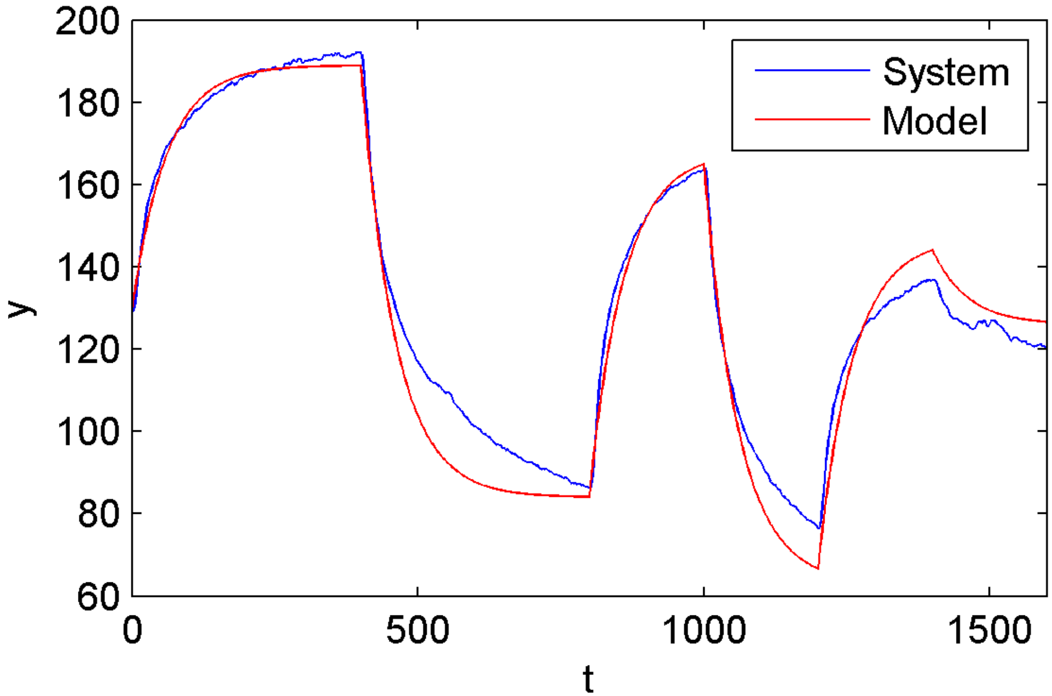
$$y(T) = y_0 + 0.632(y_{ss} - y_0) \approx 169$$

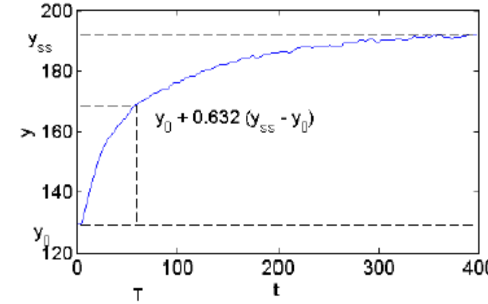
y(T)= 169ºC si rezulta **T=60s.**

In Matlab se poate modela acest sistem cu instructiunile:

```
num=[21]
den=[60 1]
H=tf(num,den)
% num si den contin coeficientii puterilor lui s in ordine descrescatoare
```

Validare functie de transfer





$$K = \frac{y_{ss} - y_0}{u_{ss} - u_0} \approx \frac{192 - 129}{9 - 6} \approx 21$$

$$y(T) = y_0 + 0.632(y_{ss} - y_0) \approx 169,$$

y(T)= 169°C si rezulta **T=60s.**

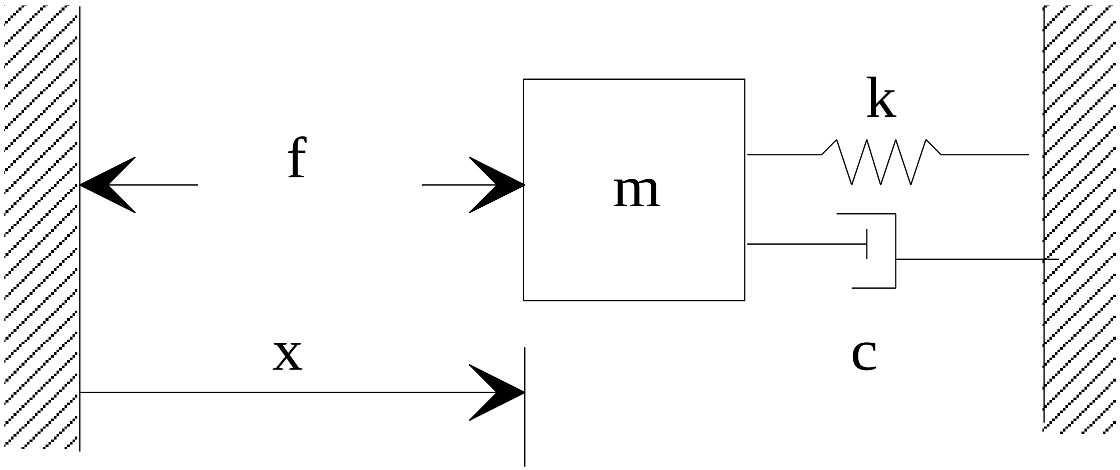
Identificarea nu este una perfecta. Dinamica racirii este mai lenta decat dinamica incalzirii, deci in realitate acest sistem nu este unul de ordinal 1.

Dar, functia de transfer identificata este suficienta pentru o prima aproximare grosiera a modelului sistemului.

Suma erorilor patratice pe datele de validare:

$$J = \frac{1}{N}\sum_{k=1}^{N} e^2(k) = \frac{1}{N}\sum_{k=1}^{N}(\hat{y}(k) - y(k))^2 \approx 62.10$$

Modele grafice – raspunsul indicial
Identificare bazata pe modelul unui sistem de ordin II



$m*d^2x/dt^2=f(t)-c*dx/dt-k*x(t)$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2+cs+k}$$

Forma generala pentru o astfel de functie de transfer:

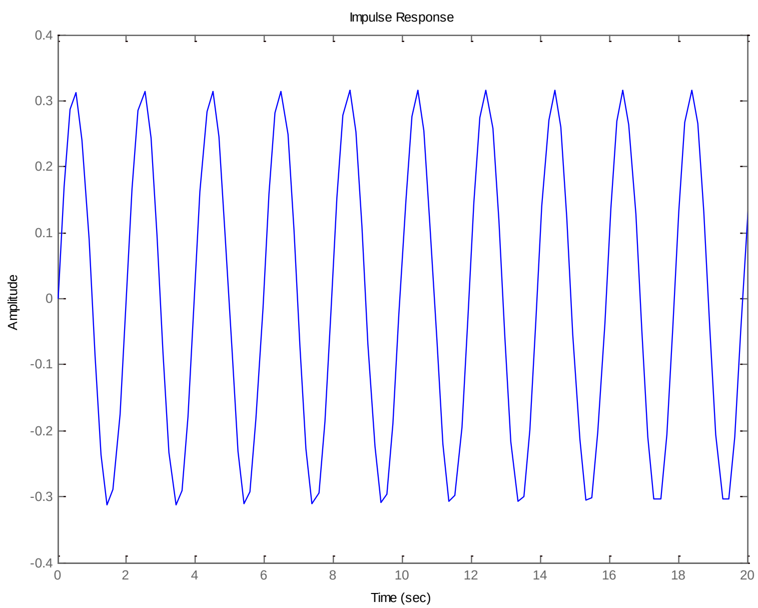
$$H(s) = \frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Unde:
K este amplificarea (K=k in exemplu)
ξ este factor de amortizare ($\xi^2= c^2/(4*k*m)$)
ω_n este pulsatia naturala (ω_n^2=k/m)

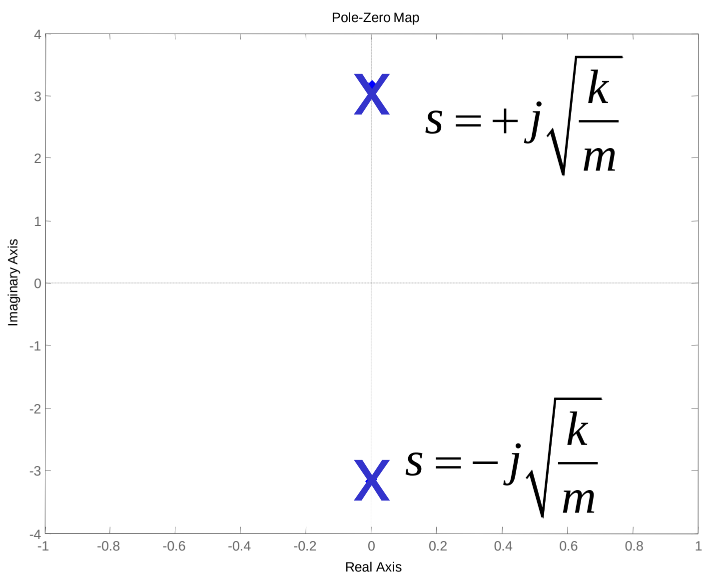
Oscilant, psi = 0

Poli cu parte reala =0 $s = \pm j\sqrt{\frac{k}{m}}$

Raspuns la impuls

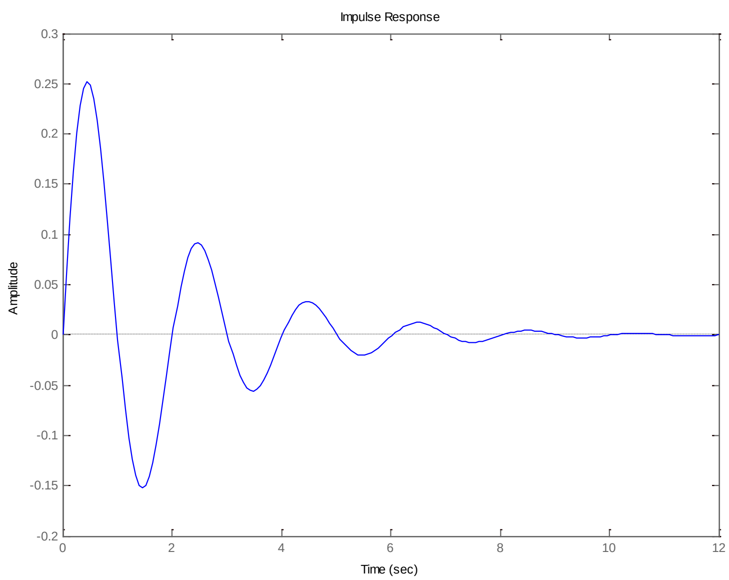


Plan poli-zerouri

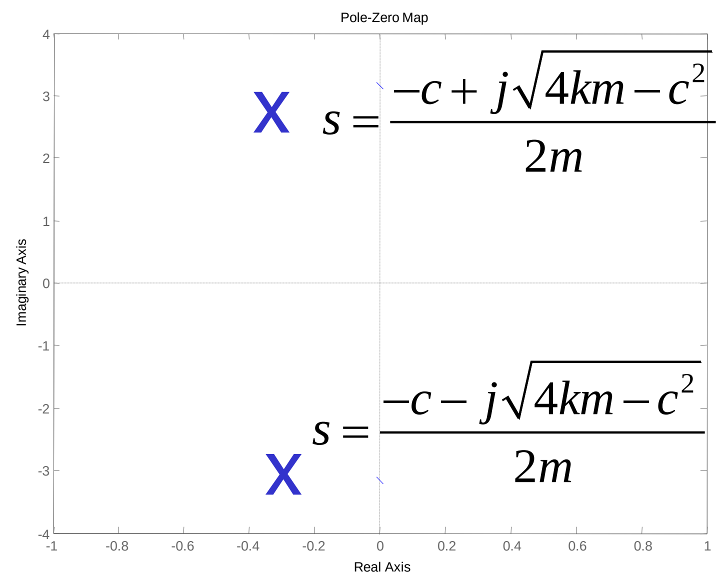


Oscilant amortizat $c^2 < 4km$

Poli complecsi: $$s = \frac{-c + j\sqrt{4km - c^2}}{2m}$$

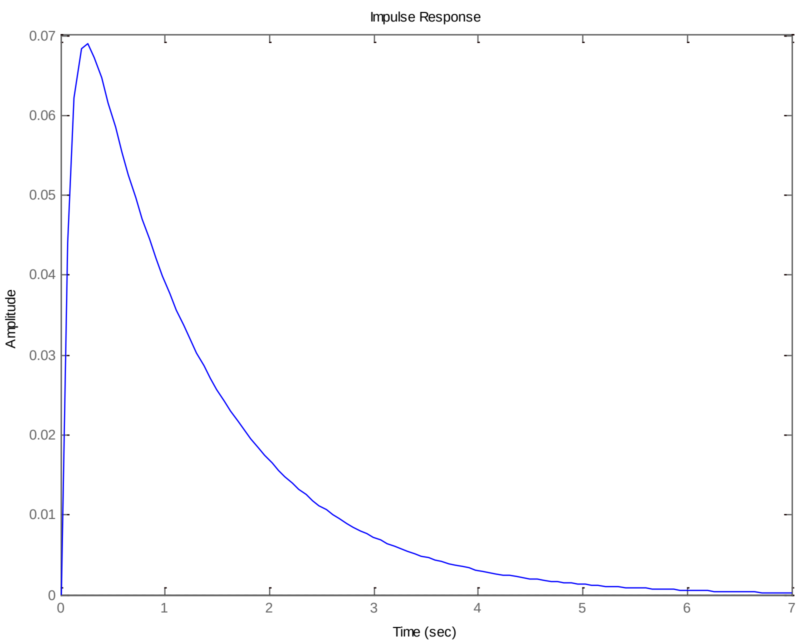


Plan poli-zerouri

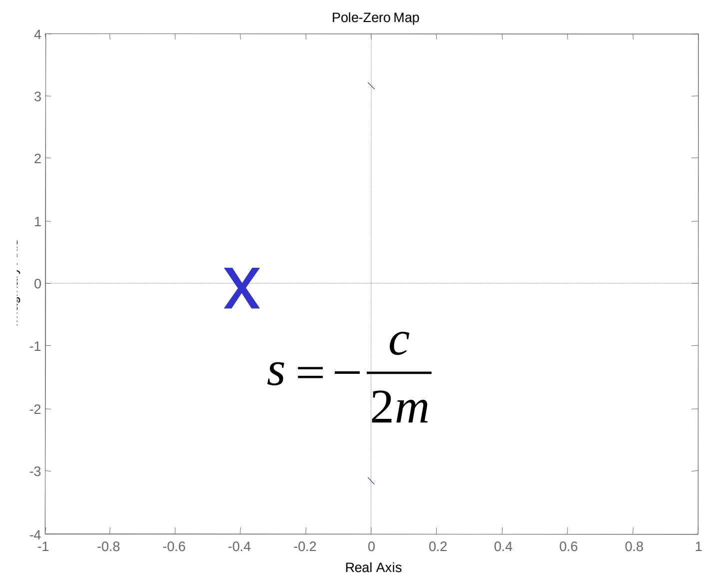


Aperiodic critic $c^2 = 4km$

Poli reali $$s = -\frac{c}{2m}$$



Plan poli-zerouri

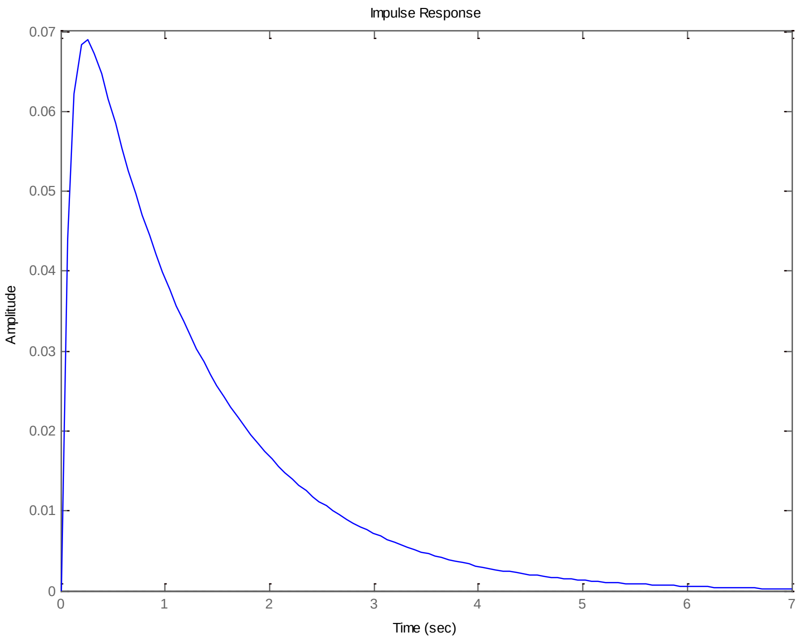


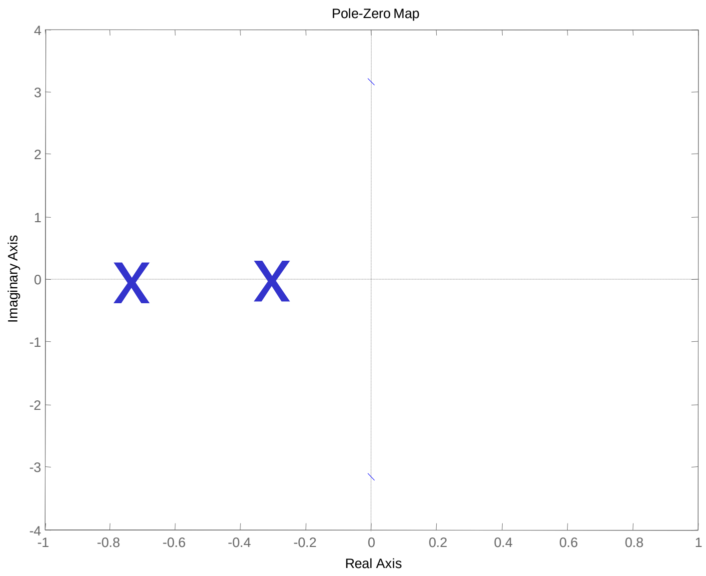
$$s = -\frac{c}{2m}$$

Aperiodic $c^2 > 4km$

Poli reali

$$s = \frac{-c + \sqrt{c^2 - 4km}}{2m}$$





Forma generala sistem de ordin II

- Forma generala

$$\frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Pulsatia naturala (rad/sec)

Factor de amortizare (subunitar)

Raspuns indicial sistem de ordin II oscilant

$$Y(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\times\frac{1}{s}=\frac{C_1}{s}+\frac{C_2}{s+\xi\omega_n-j\omega_d}+\frac{C_3}{s+\xi\omega_n+j\omega_d}$$

Unde

$$C_1=\left.\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\right|_{s=0}=1$$

$$C_2=\left.\frac{\omega_n^2}{s(s+\xi\omega_n+j\omega_d)}\right|_{s=-\xi\omega_n+j\omega_d}=\frac{-\xi}{2\sqrt{1-\xi^2}}-\frac{j}{2}=-\frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d}+j\right)$$

$$C_3=\left.\frac{\omega_n^2}{s(s+\xi\omega_n-j\omega_d)}\right|_{s=-\xi\omega_n-j\omega_d}=\frac{-\xi}{2\sqrt{1-\xi^2}}+\frac{j}{2}=-\frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d}-j\right)$$

Raspuns indicial sistem de ordin II

Raspuns indicial

Reziduuri

$$y(t) = 1 - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} + j\right)\exp\left[(-\xi\omega_n + j\omega_d)t\right] - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} - j\right)\exp\left[(-\xi\omega_n - j\omega_d)t\right]$$

$$= 1 - e^{-\xi\omega_n t}\left[\xi\frac{\omega_n}{\omega_d}\sin(\omega_d t) + \cos(\omega_d t)\right]$$

Raspuns indicial sistem de ordin II

Raspuns indicial

Poli

$$y(t) = 1 - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} + j\right)\exp[(-\xi\omega_n + j\omega_d)t] - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} - j\right)\exp[(-\xi\omega_n - j\omega_d)t]$$

$$= 1 - e^{-\xi\omega_n t}\left[\xi\frac{\omega_n}{\omega_d}\sin(\omega_d t) + \cos(\omega_d t)\right]$$

Raspuns indicial sistem de ordin II

Raspuns indicial

$$y(t) = 1 - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} + j\right)\exp\left[(-\xi\omega_n + j\omega_d)t\right] - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} - j\right)\exp\left[(-\xi\omega_n - j\omega_d)t\right]$$

$$= 1 - e^{-\xi\omega_n t}\left[\xi\frac{\omega_n}{\omega_d}\sin(\omega_d t) + \cos(\omega_d t)\right]$$

Valoare finala

Raspuns indicial sistem de ordin II

Raspuns indicial

$$y(t) = 1 - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} + j\right)\exp\left[(-\xi\omega_n + j\omega_d)t\right] - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} - j\right)\exp\left[(-\xi\omega_n - j\omega_d)t\right]$$

$$= 1 - e^{-\xi\omega_n t}\left[\xi\frac{\omega_n}{\omega_d}\sin(\omega_d t) + \cos(\omega_d t)\right]$$

Exponentiala descrecatoare

Raspuns indicial sistem de ordin II

Raspuns indicial

$$y(t) = 1 - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} + j\right)\exp\left[(-\xi\omega_n + j\omega_d)t\right] - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} - j\right)\exp\left[(-\xi\omega_n - j\omega_d)t\right]$$

$$= 1 - e^{-\xi\omega_n t}\underbrace{\left[\xi\frac{\omega_n}{\omega_d}\sin(\omega_d t) + \cos(\omega_d t)\right]}_{\text{Sinusoida}}$$

Raspuns indicial sistem de ordin II

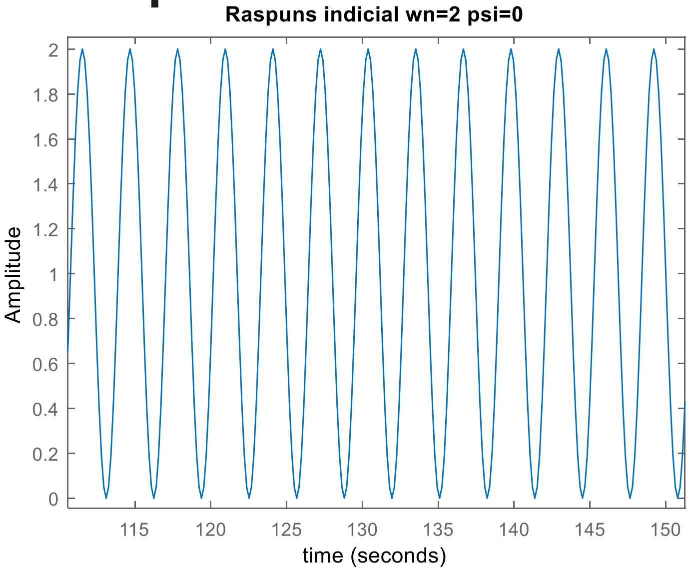
Raspuns indicial

$$y(t) = 1 - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} + j\right)\exp\left[(-\xi\omega_n + j\omega_d)t\right] - \frac{1}{2}\left(\xi\frac{\omega_n}{\omega_d} - j\right)\exp\left[(-\xi\omega_n - j\omega_d)t\right]$$

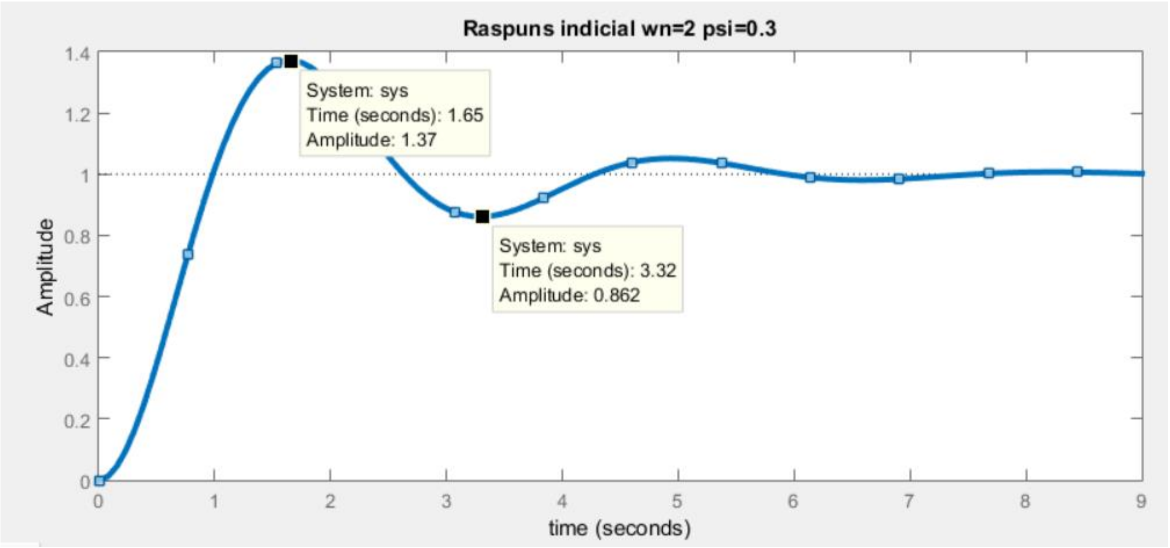
$$= 1 - \underbrace{e^{-\xi\omega_n t}\left[\xi\frac{\omega_n}{\omega_d}\sin(\omega_d t) + \cos(\omega_d t)\right]}_{\text{Sinusoida amortizata}}$$

Sinusoida amortizata

Raspuns indicial sistem ordin 2

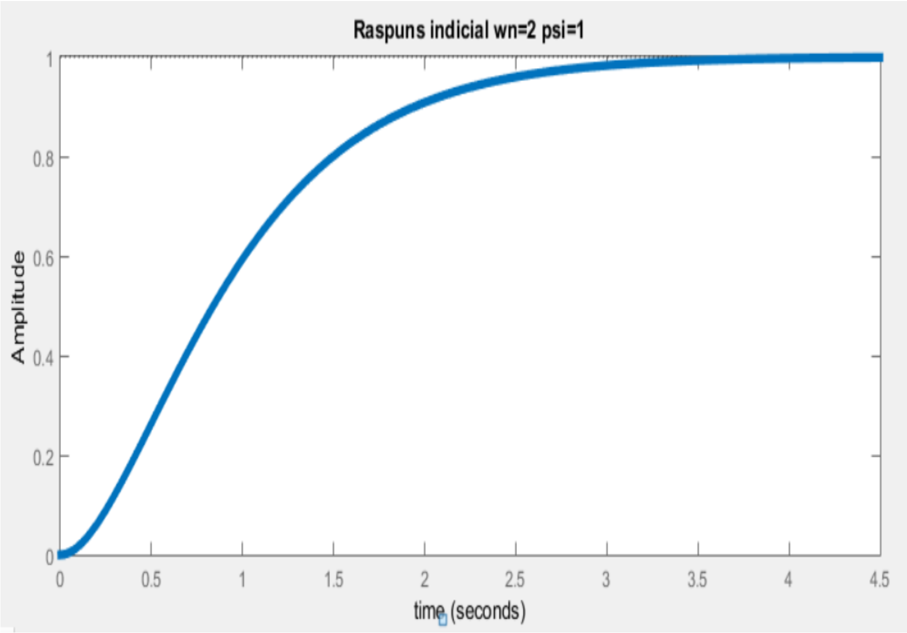


Oscilant

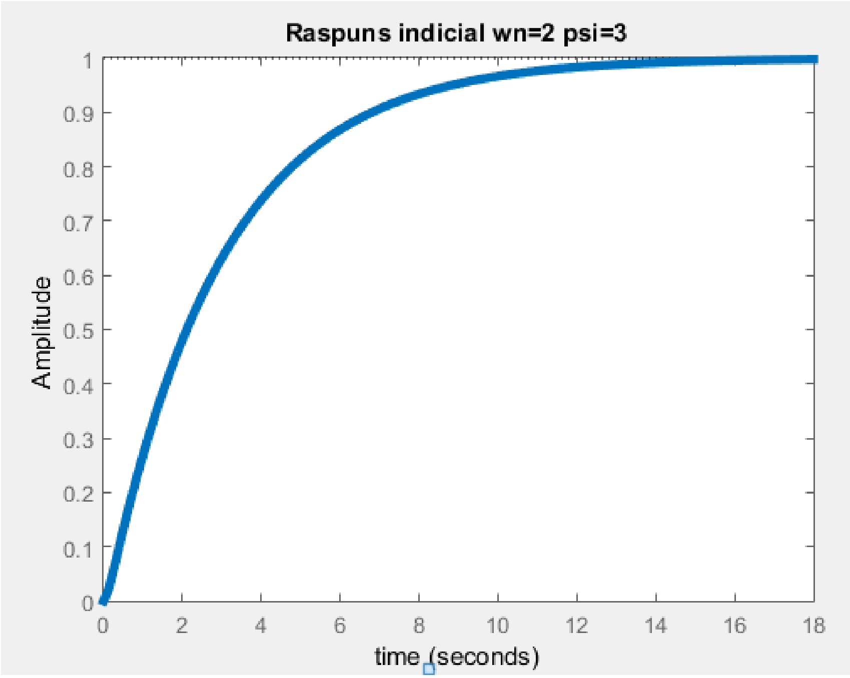


Oscilant amortizat

Raspuns indicial sistem ordin 2



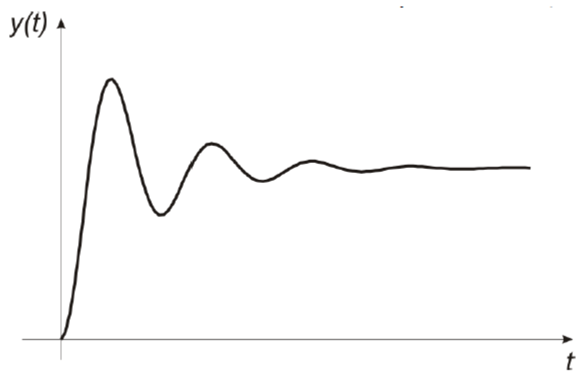
Aperiodic critic (ξ=1)



Aperiodic (ξ>1)

Raspuns indicial sistem de ordin II

Raspuns indicial (interes major pentru cazul cu oscilatii amortizate unde 0<ξ<1)

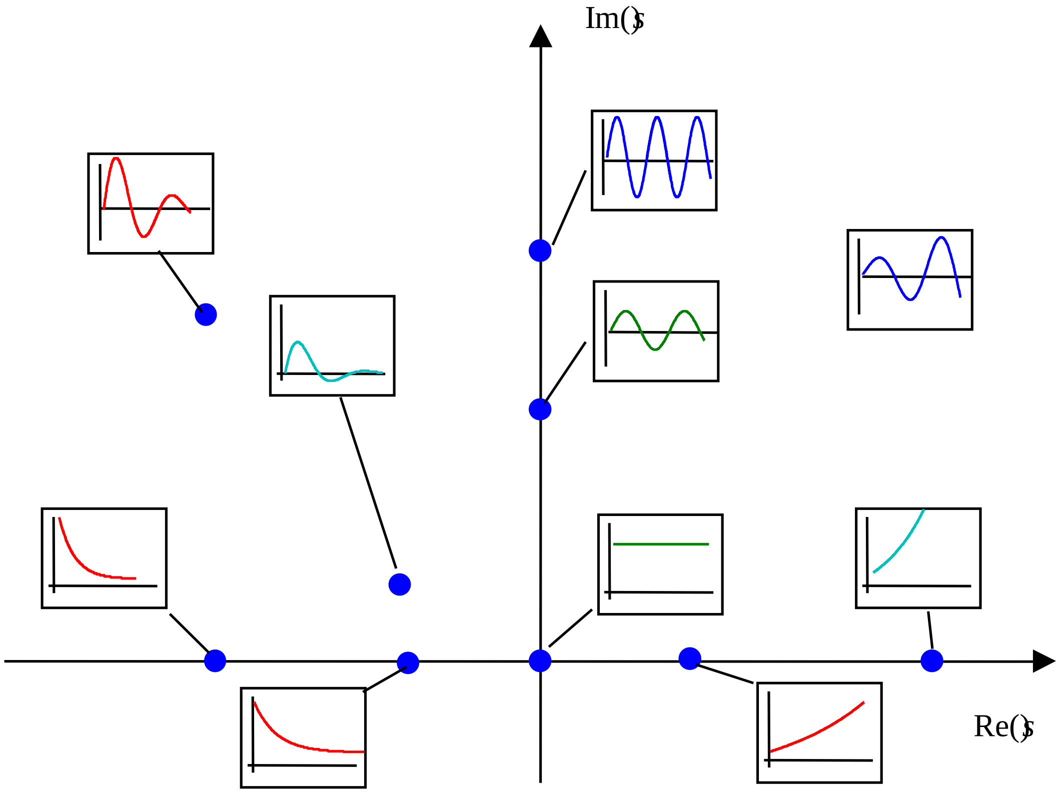


Se aduce y(t) la forma generala:

$$y(t) = K\left[1 - \frac{1}{\sqrt{1-\xi^2}} \underbrace{e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}\,t + \arccos\xi)}_{\text{Sinusoida amortizata}}\right]$$

Sinusoida amortizata

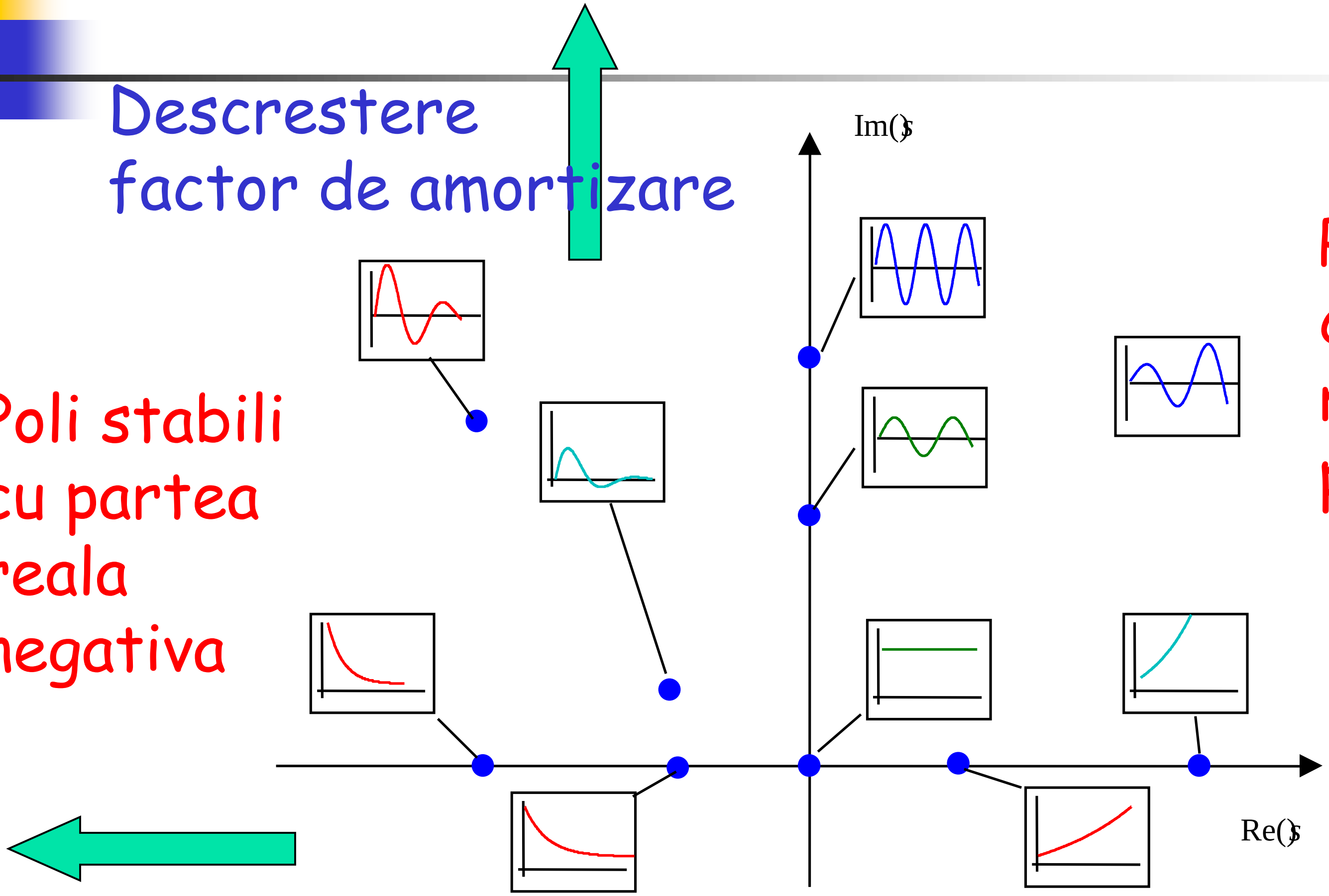
Raspunsul unui sistem in planul complex



Raspunsul unui sistem in planul complex

Descrestere
factor de amortizare

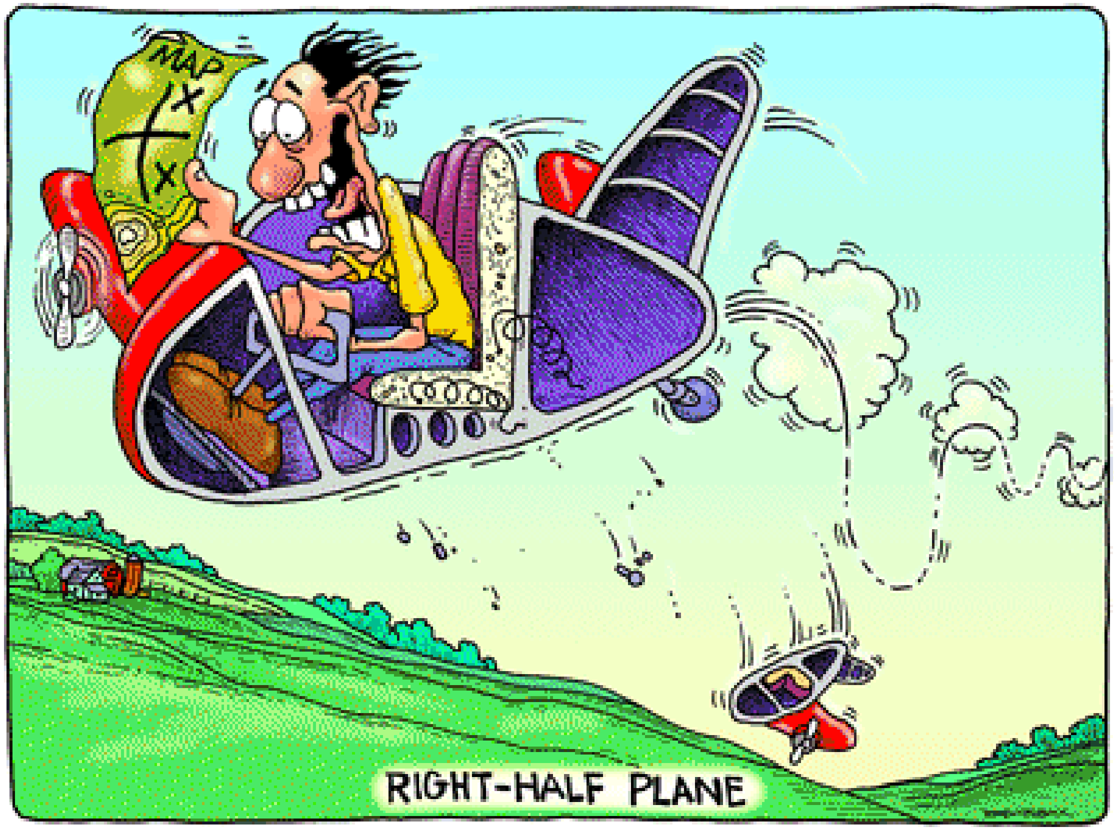
Poli stabili
cu partea
reala
negativa



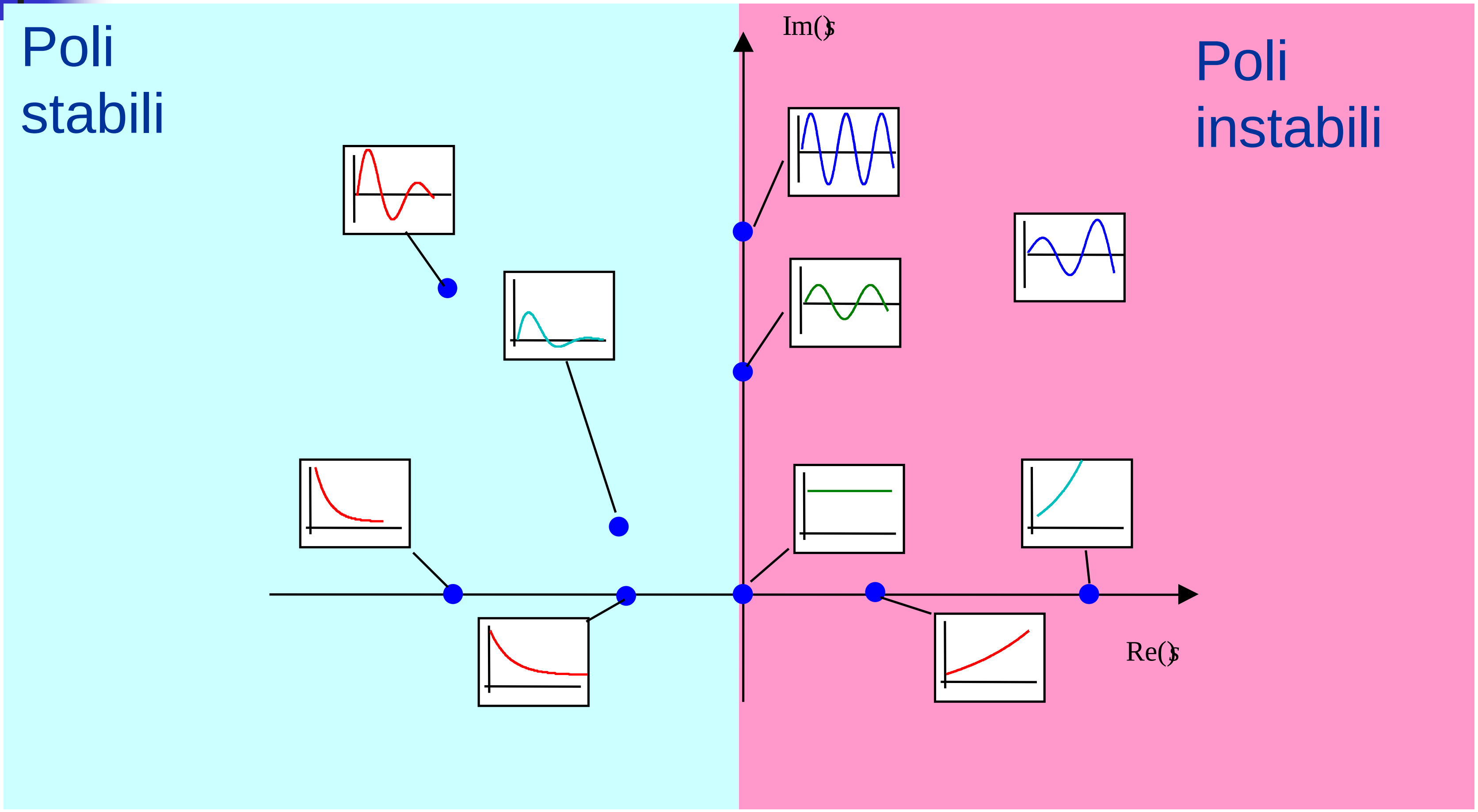
Poli instabili
cu partea
reala
pozitiva

Creste rapiditatea raspunsului

DECI NU UITATI !!!



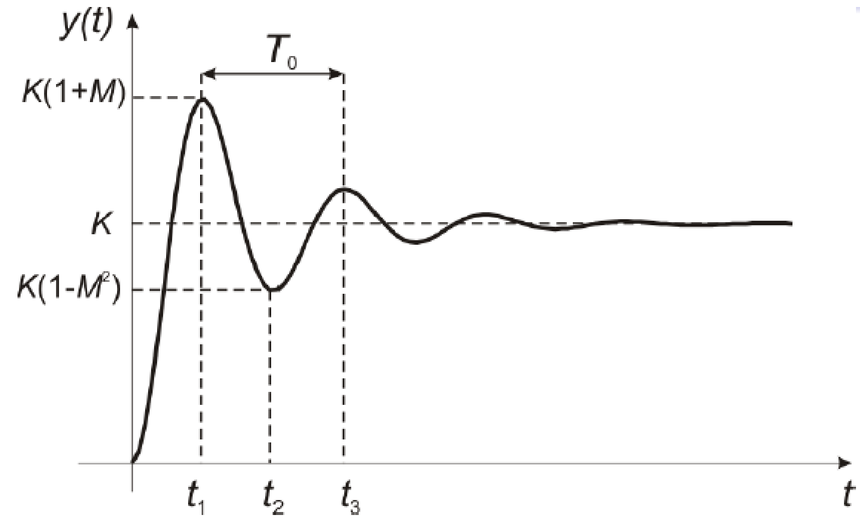
Important



Sistem de ordin II – caracteristicile raspunsului

Valoarea de regim stationar pentru y(t) se calculeaza:

$$\lim_{t\to\infty} K[1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_n\sqrt{1-\xi^2} * t\right.$$



Pentru a calcula valorile de maxim si minim la momentele t_1, t_2, t_3,… se calculeaza derivata dy/dt si se egaleaza cu zero.

$$\dot{y}(t) = \frac{K\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_n\sqrt{1-\xi^2}t) = 0$$

$$\Rightarrow t_m = \frac{m\pi}{\omega_n\sqrt{1-\xi^2}}, \quad m \geq 0 \ , m \in N$$

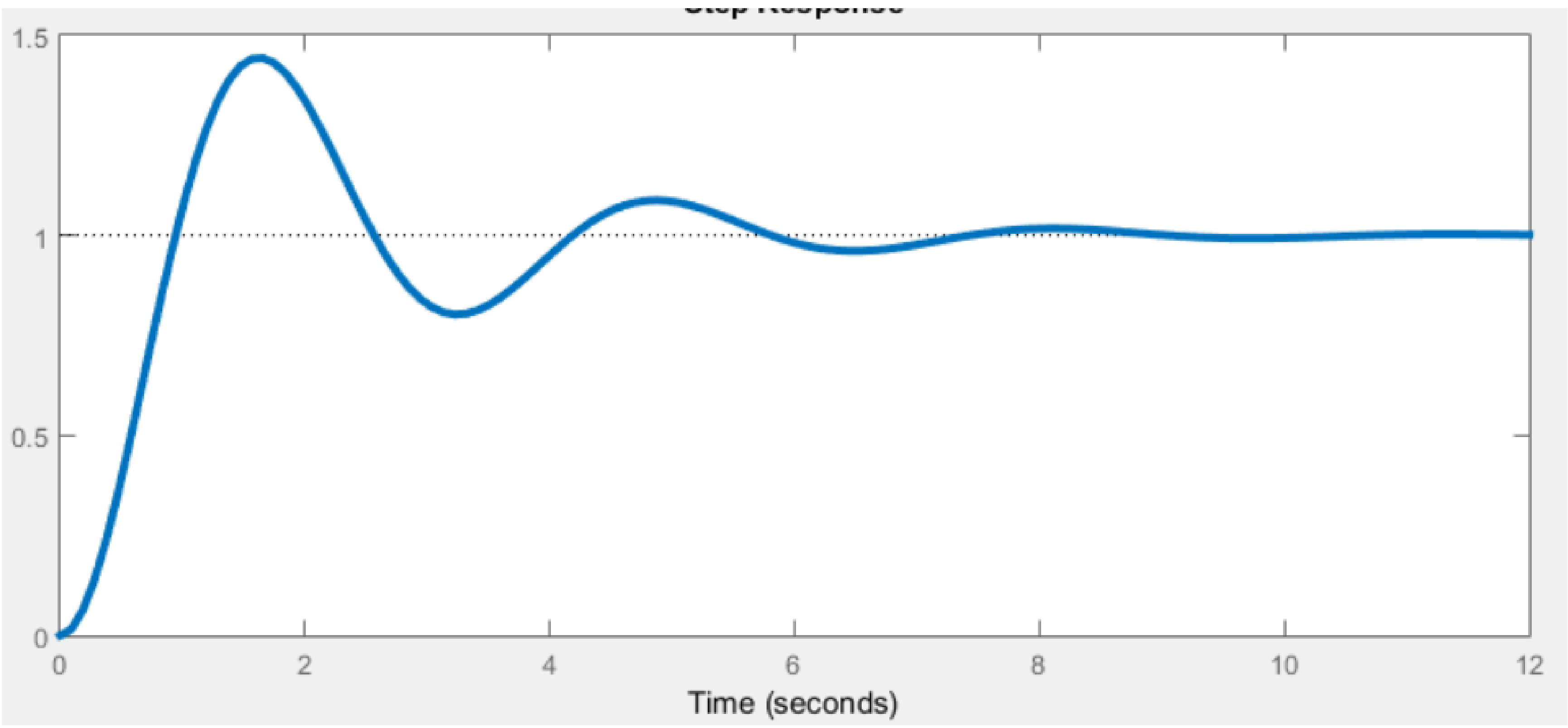
Unde: $$y(t_m) = K[1 + (-1)^{m+1} M^m]$$

Si suprareglarea M: $$M = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

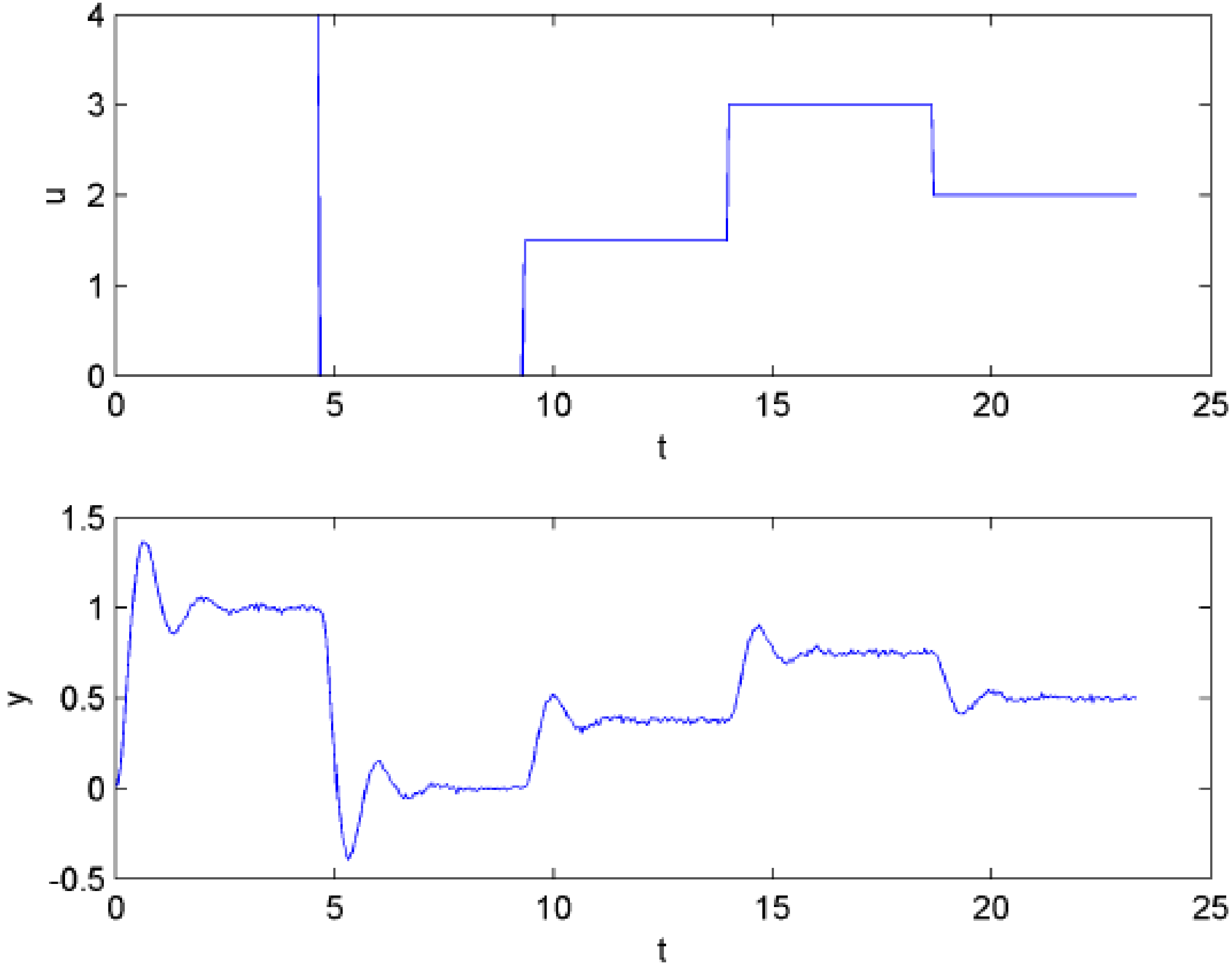
Sistem de ordin II – identificare

Un sistem real poate oferi un raspuns indicial (grafic) de tipul (model neparametric) de mai jos.

Pe baza teoriei prezentate se doreste a aproxima raspunsul cu o functie de transfer (model parametric).



Exemplu – date sistem ordin 2
(L. Busoniu, UTCj, System identification)



500 esantioane, perioada de esantionare T=0.047s.
! Zgomot de masura prezent.

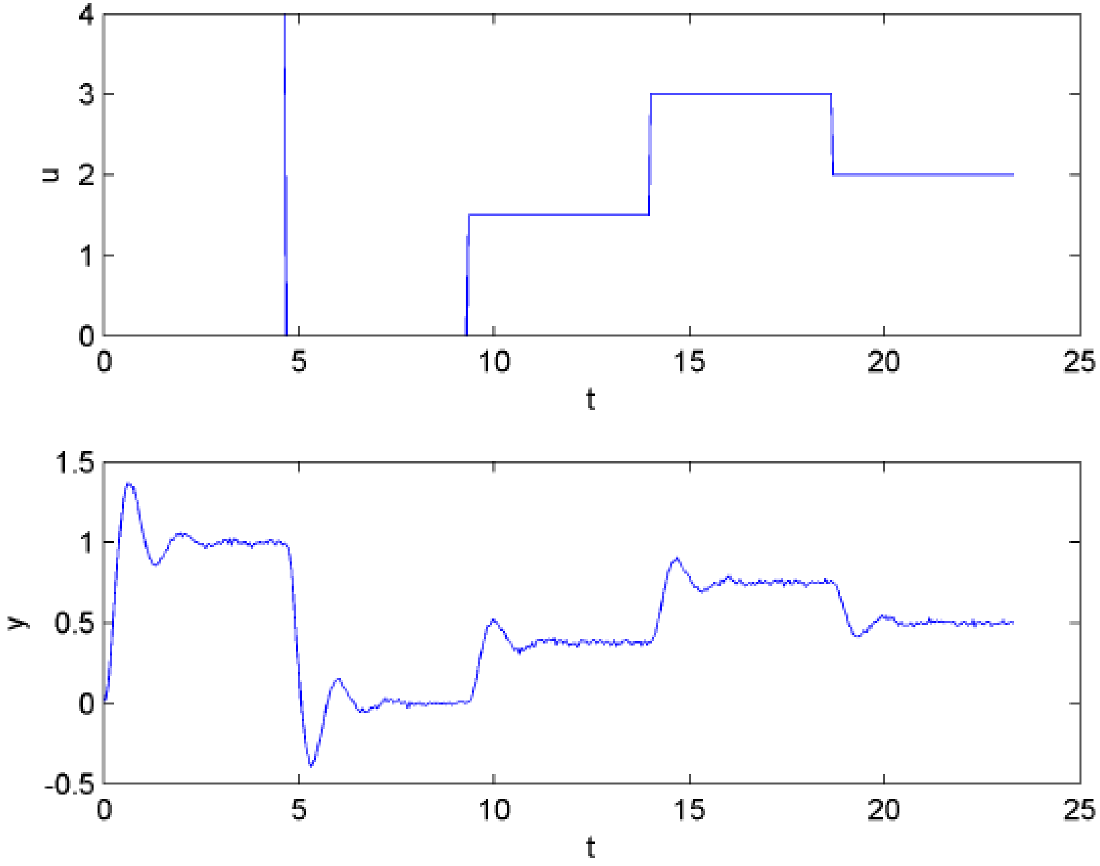
Exemplu – date sistem ordin 2

(L. Busoniu, UTCj, System identification)

Experimentul este condus din conditii initiale nule $u_0=y_0=0$.

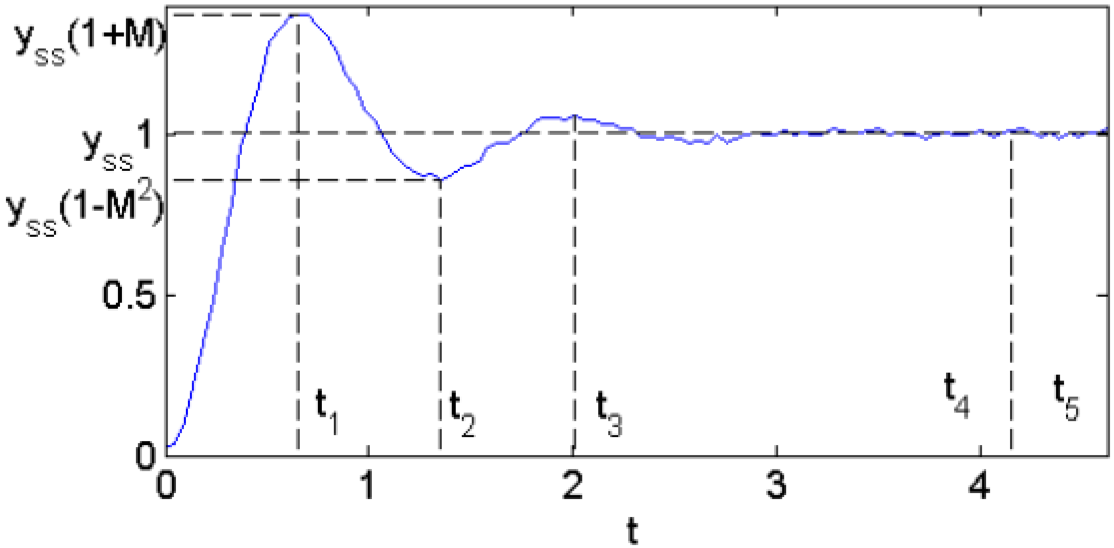
Pasii pe intrare nu sunt standard (difera de 1 unitate).

Se va utiliza primul pas pentru identificare si pasii 3-5 pentru validare. Se observa ca pasul 2 conduce sistemul in starea y=0.



Exemplu – date sistem ordin 2
(L. Busoniu, UTCj, System identification)

Modelul neparametric este grafic. Este utilizat pentru a estima o functie de transfer.



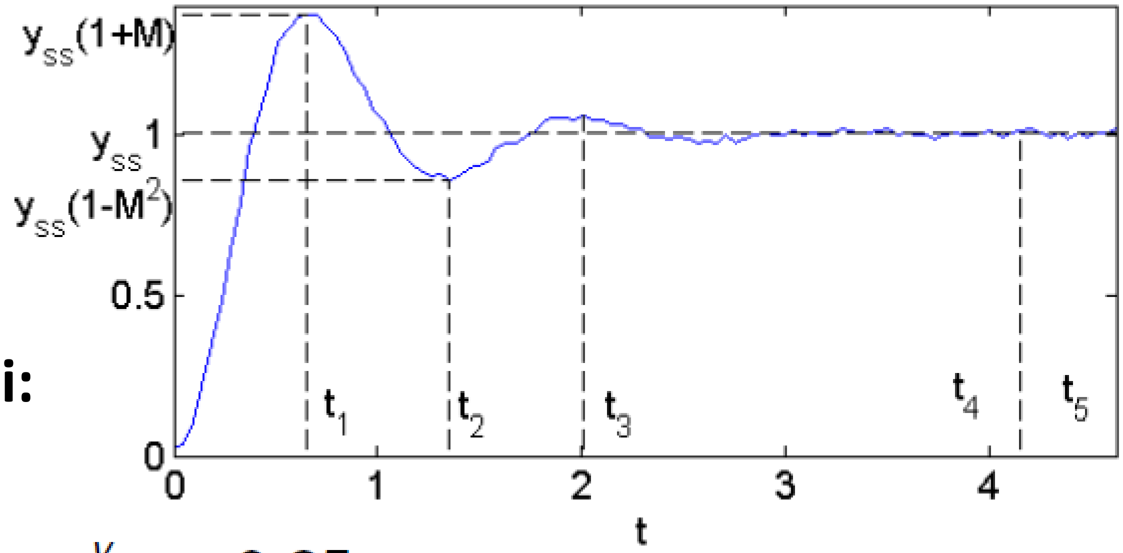
Valorile de iesire contin zgomot. Se determina valoarea de regim stationar y_{ss} ca medie a esantioanelor de la 90 la 100 (intre t_4 si t_5).

$$y_{ss} \approx \frac{1}{11} \sum_{k=90}^{100} y(k) \approx 1.00$$

Pe grafic se pot citi:

$t_1=0.65$, $t_2=1.35$, $t_3=1.96$, $y(t_1)=1.37$, $y(t_2)=0.86$ si $u_{ss}=4$

Exemplu – date sistem ordin 2
(L. Busoniu, UTCj, System identification)



Determinare parametri:

1. **Amplificare K :**

$$K = \frac{y_{ss} - y_0}{u_{ss} - u_0} = \frac{y_{ss}}{u_{ss}} \approx 0.25$$

2. **Suprareglare M:**

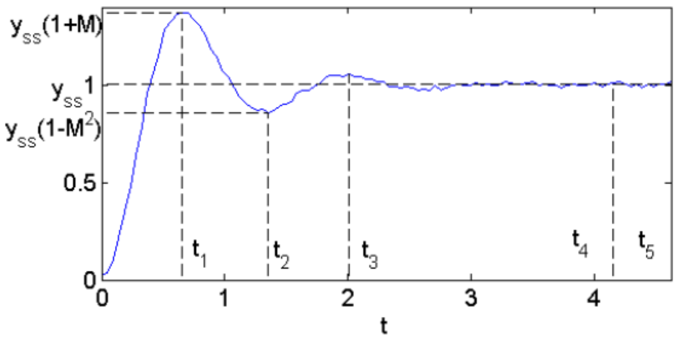
$$M = \frac{y(t_1) - y_{ss}}{y_{ss} - y_0} = \frac{y(t_1) - y_{ss}}{y_{ss}} \approx 0.36$$

3. **Factor de amortizare**

$$\xi = \frac{\log 1/M}{\sqrt{\pi^2 + \log^2 M}} \approx 0.31$$

4. **Perioada** $T_0 = t_3 - t_1 \approx 1.31$ si deci $\omega_n = \frac{2\pi}{T_0\sqrt{1-\xi^2}} \approx 5.05$

Exemplu – date sistem ordin 2
(L. Busoniu, UTCj, System identification)



Parametrii estimati:

$$\widehat{K} = 0.25$$

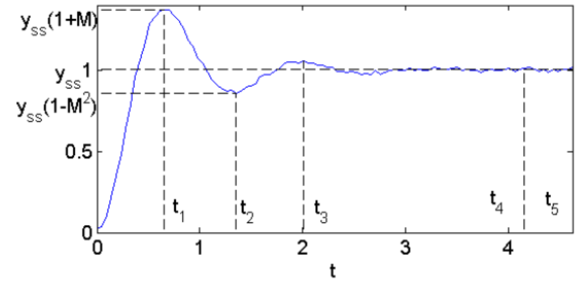
$$\widehat{\xi} = 0.31$$

$$\widehat{\omega_n} = 5.05$$

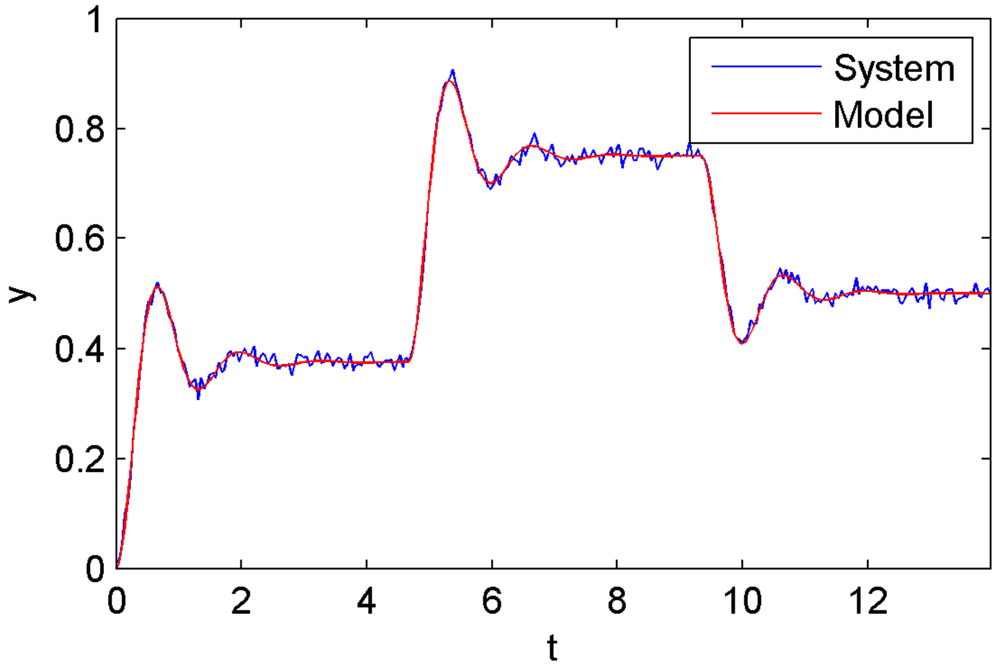
Conduc la functia de transfer:

$$\widehat{H}(s) = \frac{\widehat{K}\widehat{\omega}_n^2}{s^2 + 2\widehat{\xi}\widehat{\omega}_n s + \widehat{\omega}_n^2} = \frac{6.38}{s^2 + 3.09s + 25.51}$$

Exemplu – date sistem ordin 2
(L. Busoniu, UTCj, System identification)



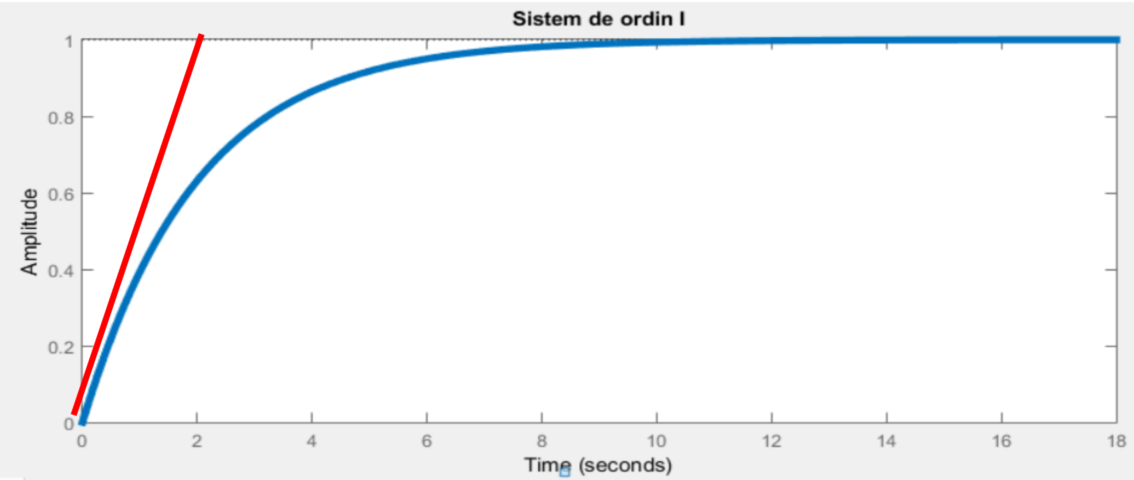
Validare cu datele de validare:

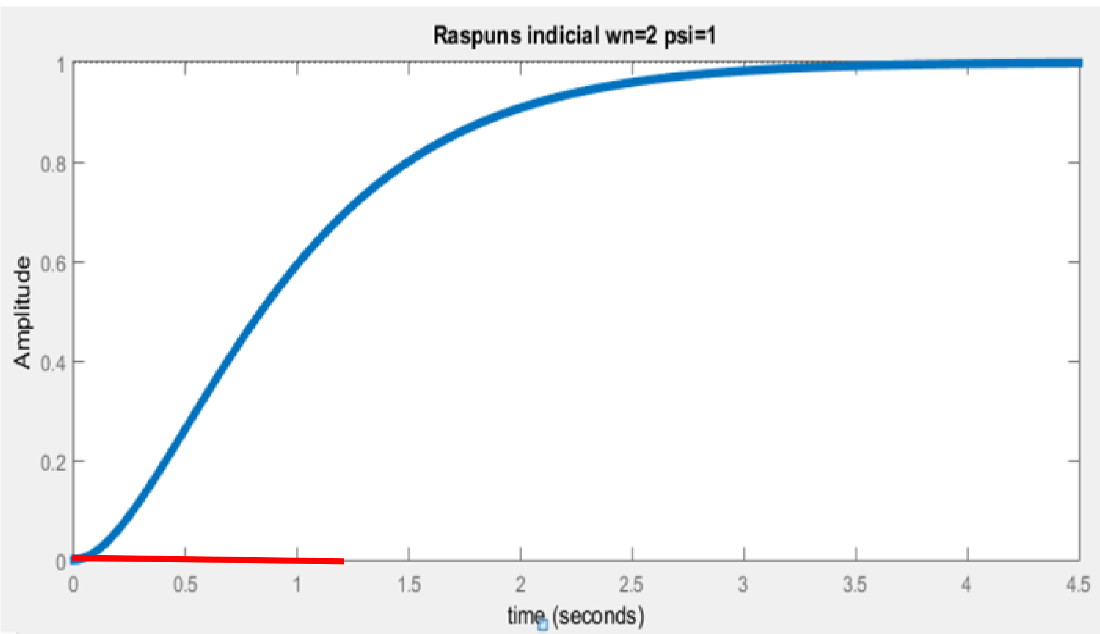


Suma erorilor patratice:

$$J = \frac{1}{N}\sum_{k=1}^{N} e^2(k) = \frac{1}{N}\sum_{k=1}^{N}(\widehat{y}(k) - y(k))^2 \approx 9.66 \cdot 10^{-5}$$

Alegerea gradului functiei de transfer (ordin sistem)





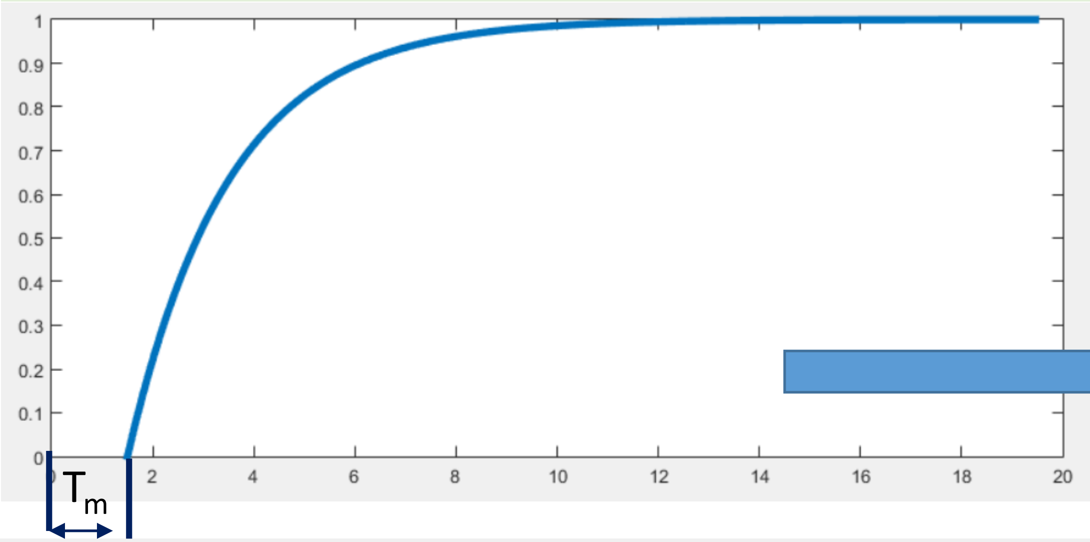
Cu rosu – derivata la t=0;

Pentru sistemul de ordin II, chiar daca este aperiodic critic, derivata este 0 la t=0 (reprezentare ca tangenta la axa timpului).

Pentru sistemul de ordin I panta tangentei este K/T.

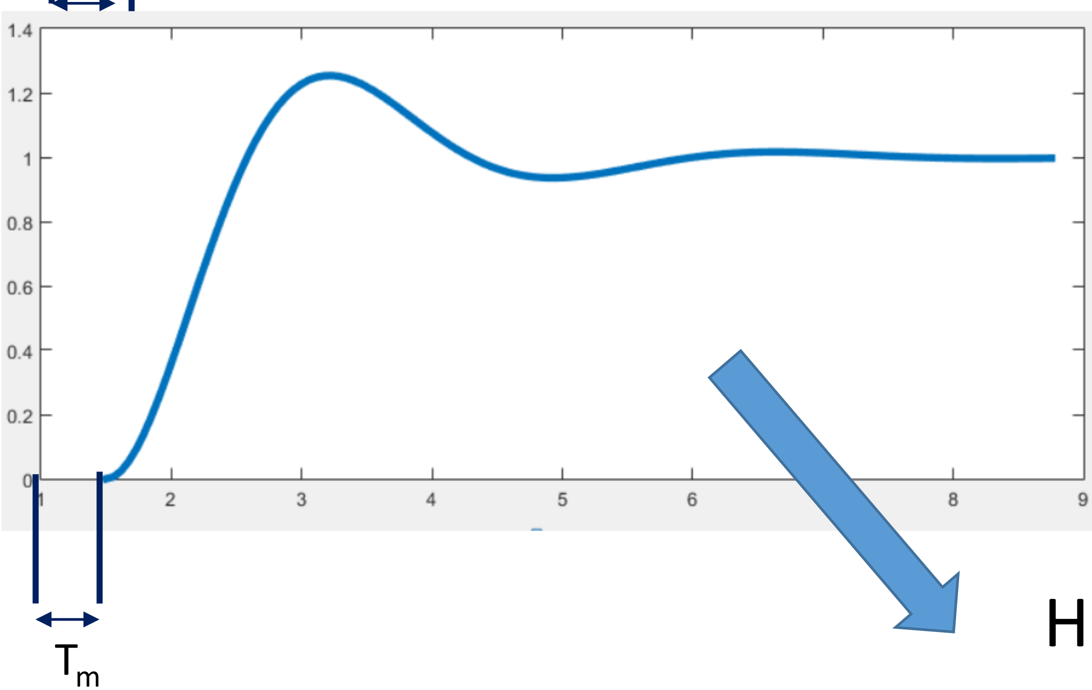
Aceasta este o informatie care permite alegerea corecta a ordinului sistemului.

Sisteme cu timp mort (T_m)



De pe grafic se citeste timpul mort T_m. Se scriu functiile de transfer:

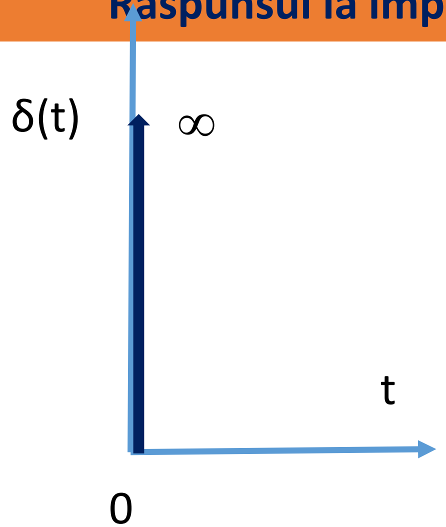
$$H(s) = \frac{K}{sT+1} e^{-sT_m}$$

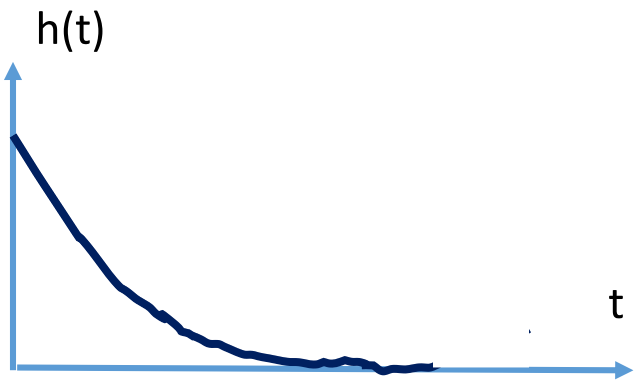


$$H(s) = \frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} e^{-sT_m}$$

Modele grafice – raspunsul la impuls

Raspunsul la impuls (Dirac, ideal)





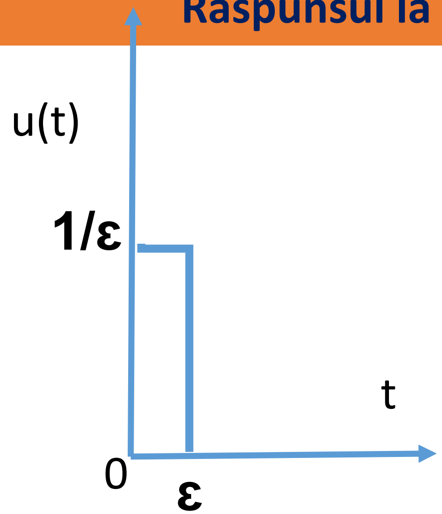
$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

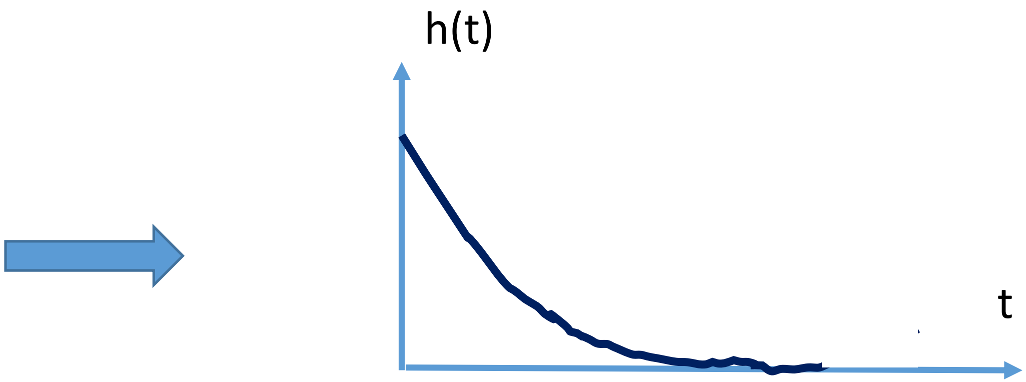
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

In fapt, impulsul ideal nu este o functie si necesita notiunea de *distributii* pentru a fi definit.

Modele grafice – raspunsul la impuls

Raspunsul la impuls (real)





$$u(t) = \begin{cases} \frac{1}{\varepsilon}, & t \in [0, \varepsilon) \\ 0, & t \notin [0, \varepsilon) \end{cases}$$

$$\int_{-\infty}^{\infty} u(t)dt = 1$$

Aria dreptunghiului este 1.

ε << decat constantele de timp din sistem.

Aplicarea impulsului real introduce erori (de analiza) fata de cel ideal, dar daca ε este mic, aceste erori sunt acceptabile.

Modele grafice – raspunsul la impuls

Raspunsul la impuls (amintire de la TS)

Definitie: **Raspunsul la impuls** al unui sistem monovariabil neted, **numit functie sau distributie pondere**, **este originalul *h(t)* corespunzator functiei de transfer *H(s)***

$$y(t) = L^{-1}\{ H(s) \bullet 1 \} = (h \bullet \delta) = \int_0^t h(t-\sigma)\delta(\sigma)\,d\sigma =$$
$$= h(t) = L^{-1}\{ H(s) \} \qquad (1)$$

Dar raspunsul indicial este: $w(t) = L^{-1}\{W(s)\} = L^{-1}\left\{\frac{H(s)}{s}\right\}$

de unde: $Y_{raspuns\ indicial}(s) = \frac{1}{s} Y_{raspuns\ impuls}(s)$

si deci: $y_{raspuns\ indicial}(t) = \int_0^t y_{raspuns\ impuls}(\tau)d\tau$

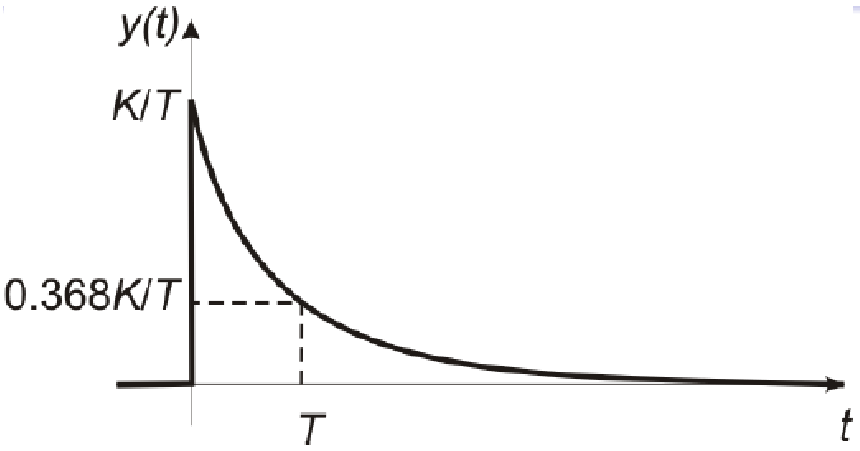
$$y_{raspuns\ impuls}(t) = \frac{dy_{raspuns\ indicial}(t)}{dt}$$

Raspunsul la impuls este derivata raspunsului indicial

Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin I) partea teoretica

$$H(s) = \frac{K}{Ts+1}$$



Se rezolva relatia (1) si se deduce h(t)=y(t).

$$h(t) = y(t) = \frac{K}{T}e^{-t/T} \ , t \geq 0$$

de unde:

$$h(0) = \frac{K}{T} = y_{max}$$

$$h(T) = \frac{K}{T}e^{-1} = y_{max}e^{-1} \approx 0.367879 * y_{max}$$

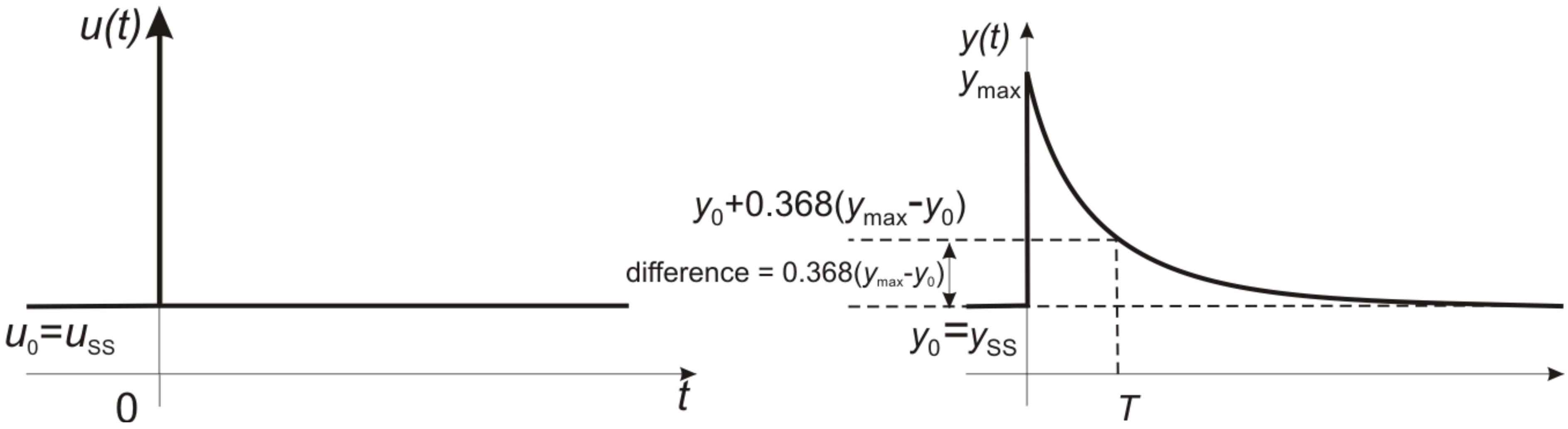
Raspunsul este in stare stabila dupa aproximativ 4T, pentru care se calculeaza:

$$h(4T) = \frac{K}{T}e^{-4} = y_{max}e^{-4} \approx 0.0183156 * y_{max}$$

Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin I) partea teoretica

Cand conditiile initiale sunt diferite de zero, raspunsul este decalat cu conditia initiala y_0 (u_0 pe intrare) pe axa ordonatelor.



Comportarea sistemului este descrisa de:

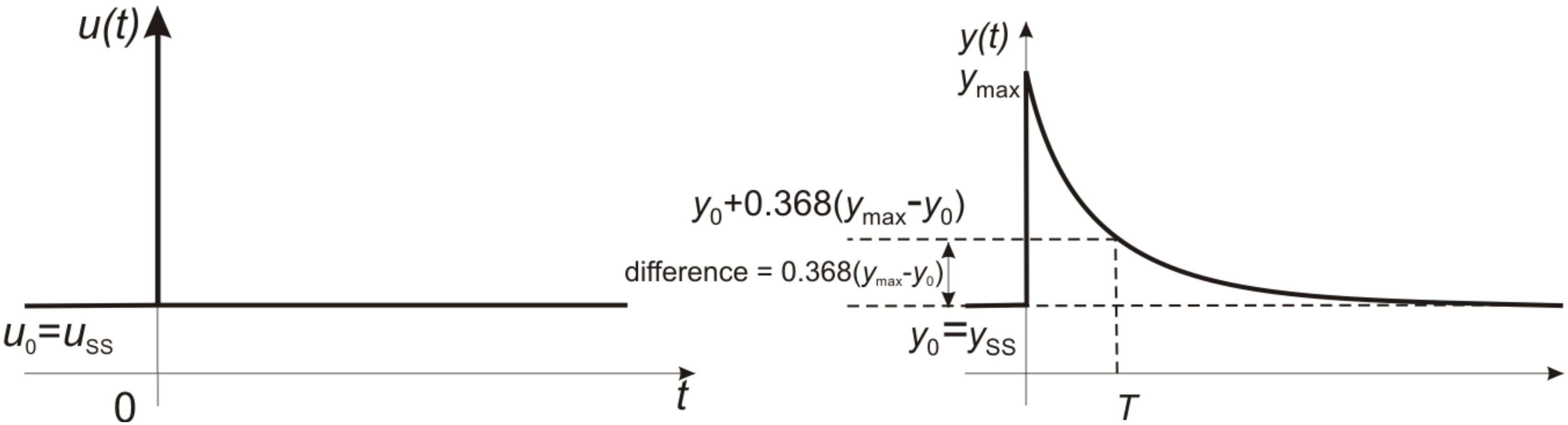
$$y_{max} = y_0 + \frac{K}{T}$$

$$y(T) = y_0 + 0.368(y_{max} - y_0)$$

Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin I) : determinarea parametrilor raspunsului real

Raspunsul dat grafic este un model neparametric. Este utilizat pentru a deduce un model parametric tip functie de transfer.



Consideram conditii initiale nenule. Se efectueaza pasii:

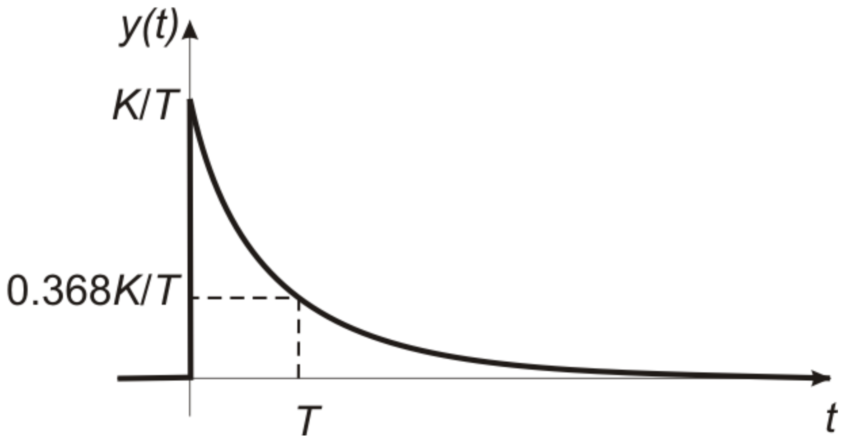
1. Citim valorile initiale si de regim stabilizat: $y_0=y_{ss}$ si $u_0=u_{ss}$, de unde $K=y_{ss}/u_{ss}$
2. Se citeste de pe grafic y_{max}, identificam valoarea timpului pentru care raspunsul ajunge la $0.368(y_{max}-y_0)$ si astfel valoarea timpului T.
3. Cu K si T determinate se scrie functia de transfer: $H(s) = \frac{K}{Ts+1}$

Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin I) : determinarea parametrilor raspunsului real

Cazul conditiilor initiale nule

Se poate estima amplificarea K utilizand valoarea lui y_{max}, unde y_{max}=K/T, dar in practica determinarea nu va fi una foarte precisa (zgomot de masura, semnalul tip impuls nu este ideal).



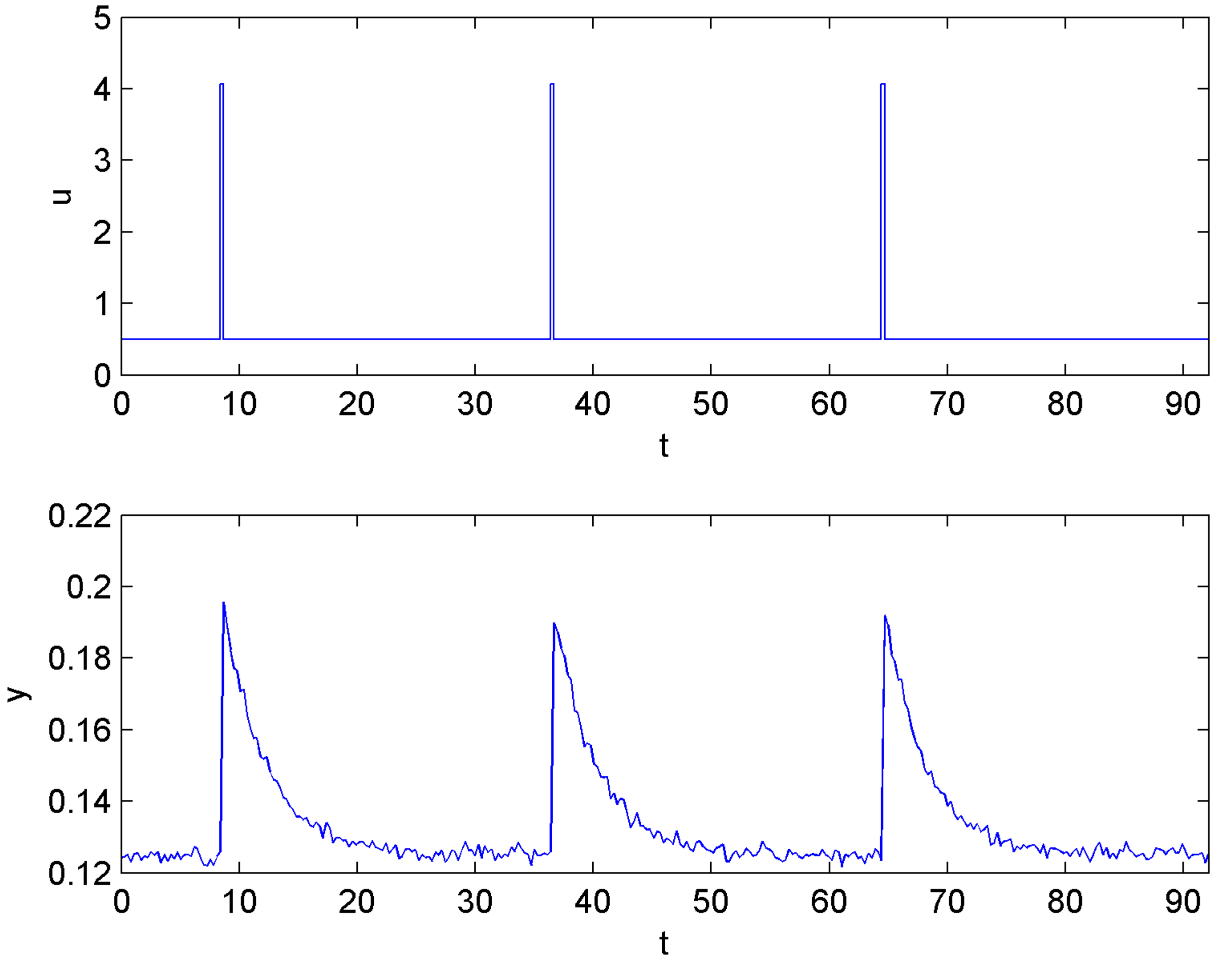
1. Se citeste de pe grafic y_{max}, identificam valoarea timpului pentru care raspunsul ajunge la 0.368*y_{max} si astfel valoarea timpului T;
2. Se determina K=y_{max}*T;
3. Cu K si T determinate se scrie functia de transfer: $H(s) = \frac{K}{Ts+1}$

Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor (exp. L Busoniu, System Identification UTCj)

300 esantioane; perioada de esantionare T_e=0.28 s (primele 30 esantioane sunt din partea de regim stabilizat, si apoi cate 100 pentru fiecare raspuns la impuls).

Impulsurile au $\varepsilon = T_e = 0.28$s si amplitudinea $1/\varepsilon \approx 3.57$.



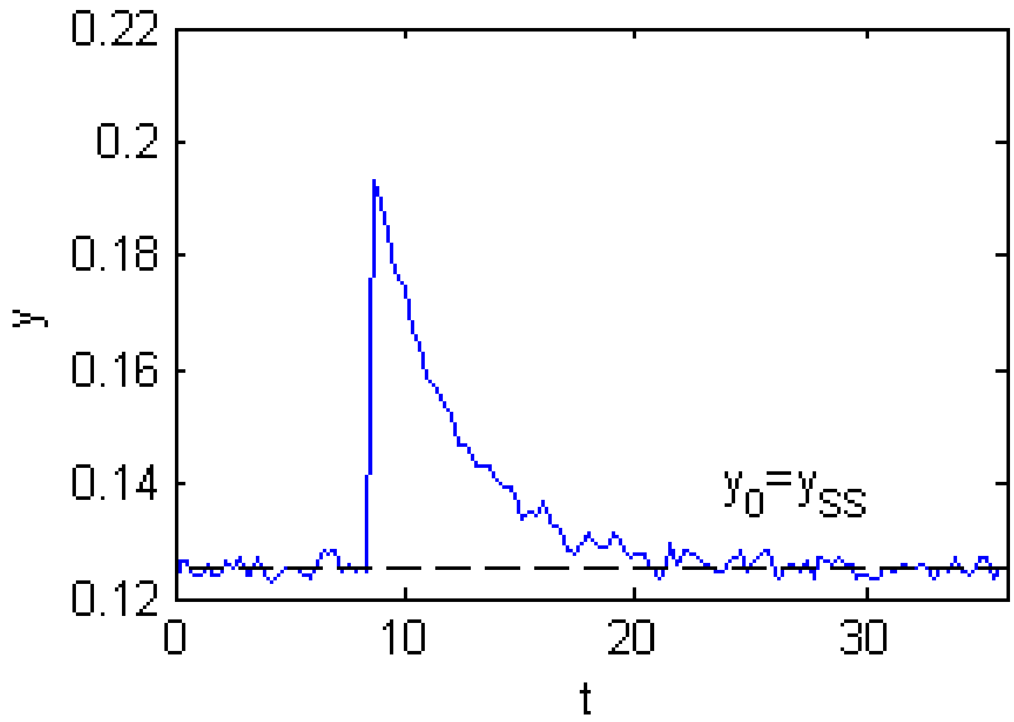
Identificare: primul impuls; Validare: impulsurile 2 si 3

Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor (exp. L Busoniu, System Identification UTCj)

Se utilizeaza un grafic(model neparametric) si se va estima o functie de transfer (model parametric).

$$u_0 = u_{ss} = 0.5$$



Pentru a gasi y_{ss} se face o medie pe cateva esantioane:

$$y_{ss} = y_0 \approx \frac{1}{11} \sum_{k=120}^{130} y(k) \approx 0.13$$

Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor (exp. L Busoniu, System Identification UTCj)

Valoarea maxima y_{max}≈0.19 este atinsa la t_1 ≈8.86s.

Valoarea

$$y_0 + 0.368(y_{\max} - y_0) \approx 0.15$$

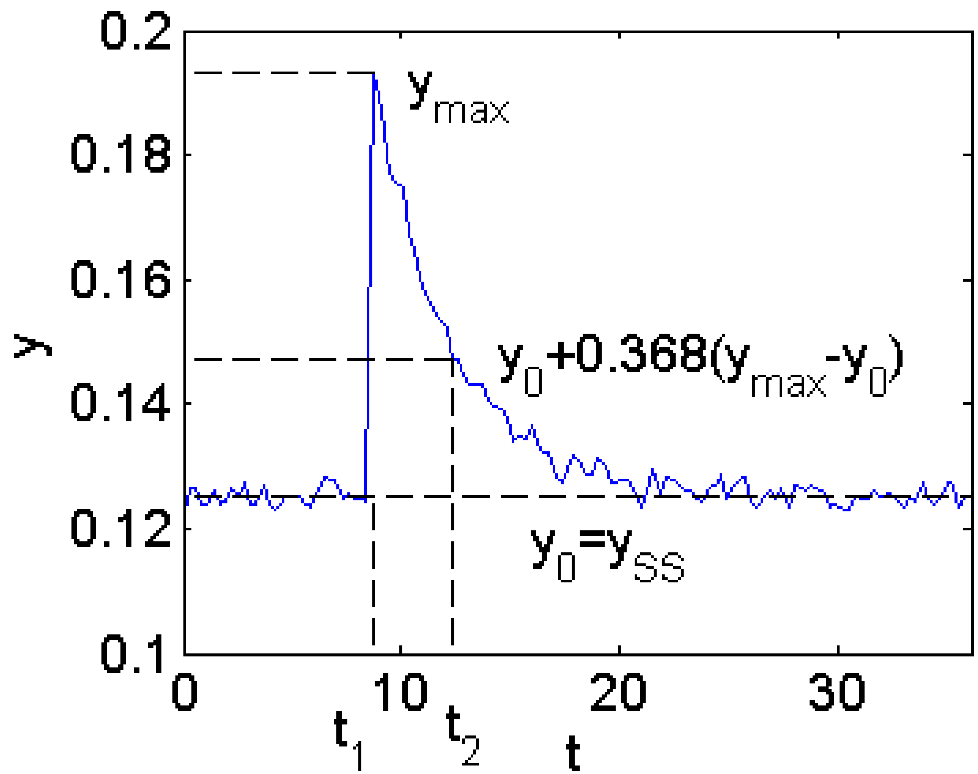
se atinge la t_2 ≈12.60s.

Rezulta:

$$K = y_{ss}/u_{ss} \approx 0.25$$

$$T = t_2 - t_1 \approx 3.92s$$

*Pentru calculul T s-a luat in considerare t_1 (diferit de zero), momentul la care este aplicat impulsul.



H(s) estimata:

$$\widehat{H}(s) = \frac{\widehat{K}}{\widehat{T}s + 1} = \frac{0.25}{3.92s + 1}$$

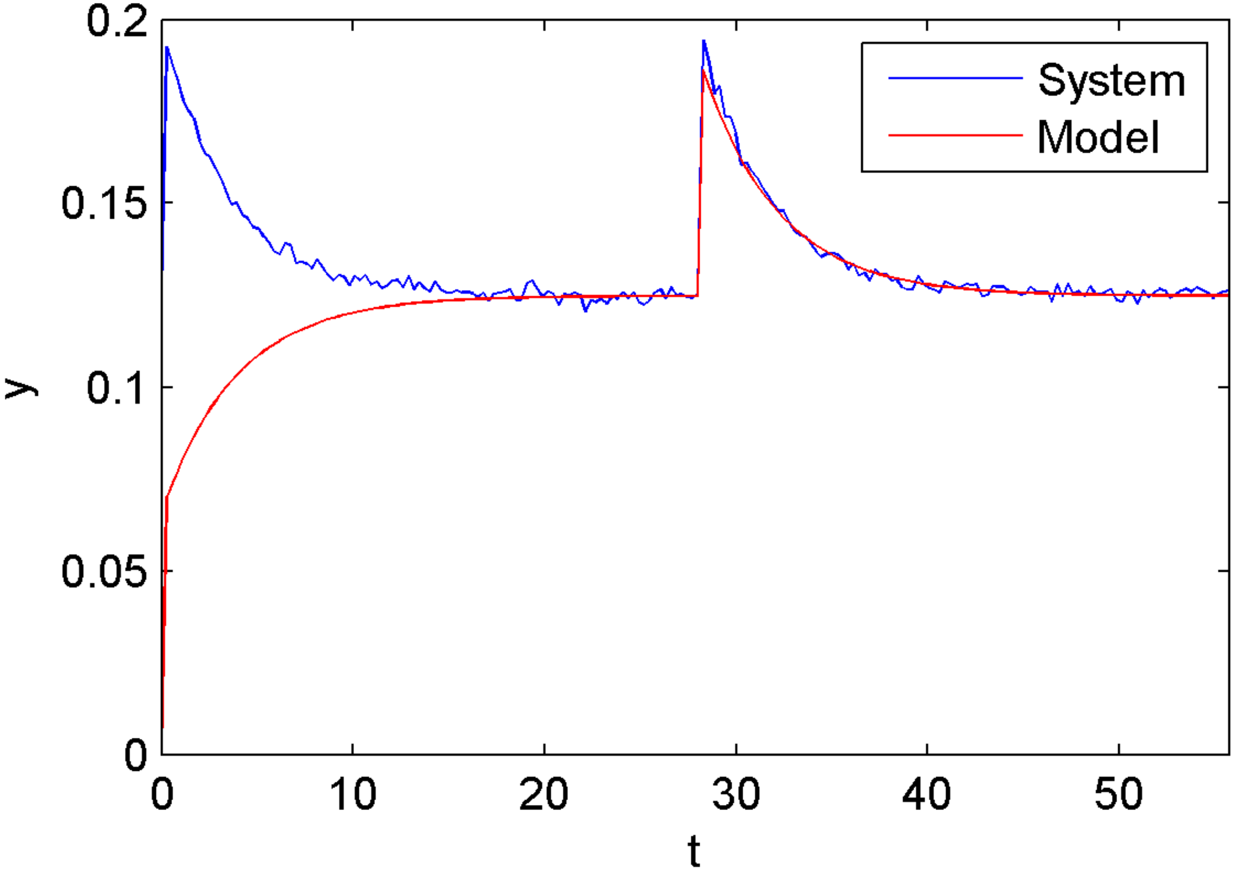
Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor (exp. L Busoniu, System Identification UTCj)

Validarea modelului tip functie de transfer.

Se compara datele reale (impulsurile 2 si 3) cu cele ale modelului.

Simularea nu ia in calcul conditiile initiale diferite de zero, de aceea prima parte prezinta diferente mari.



Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor (exp. L Busoniu, System Identification UTCj)

Pentru o simulare care ia in calcul si conditiile initiale:

- Trecem la modelul in spatiul starilor:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

- Daca functia de transfer era:

$$H(s) = \frac{K}{Ts+1} = \frac{Y(s)}{U(s)}$$

- Se trece in domeniul timp: $\dot{y}(t) = \frac{-1}{T}y(t) + \frac{K}{T}u(t)$

- Se adopta variabila de stare *x=y* (sistem de ordin I)

$$\dot{x}(t) = -\frac{1}{T}x(t) + \frac{K}{T}u(t)$$
$$y(t) = x(t)$$

- Si modelul in spatial starilor are matricile:

$$A = -\frac{1}{T},\ B = \frac{K}{T},\ C = 1,\ D = 0$$

Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor (exp. L Busoniu, System Identification UTCj)

Pentru o simulare care ia in calcul si conditiile initiale:

- Trecem la modelul in spatiul starilor:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

- Modelul estimat este:

$$\dot{x}(t) = -\frac{1}{\widehat{T}}x(t) + \frac{\widehat{K}}{\widehat{T}}u(t) = -0.26x(t) + 0.06u(t)$$
$$y(t) = x(t)$$

- In Matlab se va incarca modelul estimat cu instructiunea:

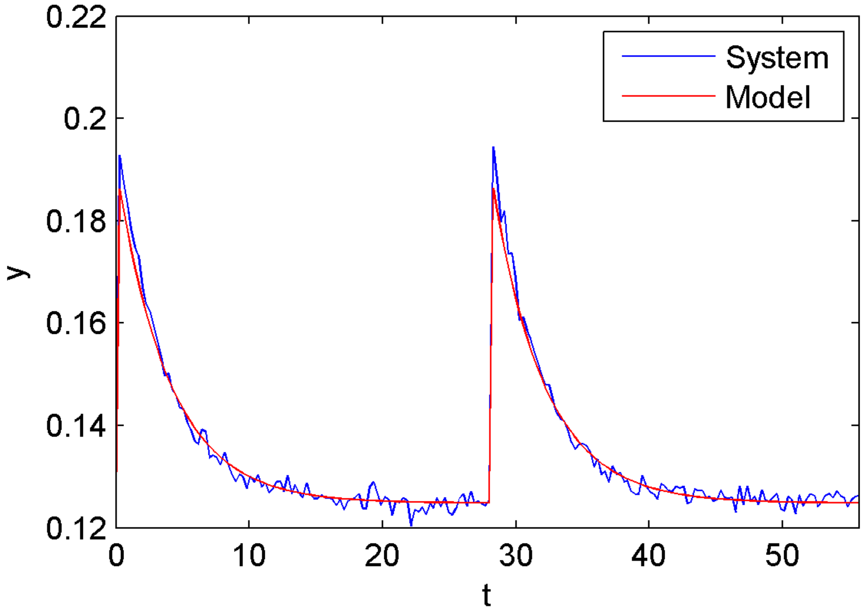
- $$H_{estimat} = ss(A, B, C, D)$$

Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor (exp. L Busoniu, System Identification UTCj)

Pentru o simulare care ia in calcul si conditiile initiale:

- Pentru a lua in considerare si conditiile initiale, in simulare, se va considera $x(0)=y_0$ la initierea simularii.



- Suma erorilor patratice pe datele de validare:

$$J = \frac{1}{N}\sum_{k=1}^{N} e^2(k) \approx 3.74 \cdot 10^{-6}$$

Modele grafice – raspunsul la impuls

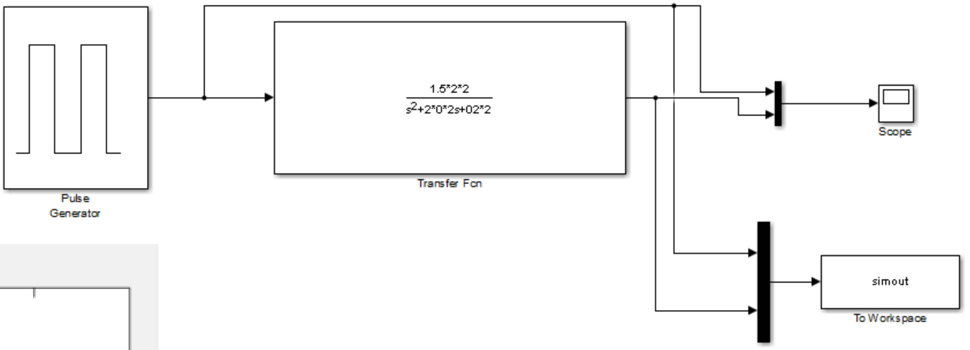
Raspunsul la impuls (sistem de ordin II) partea teoretica

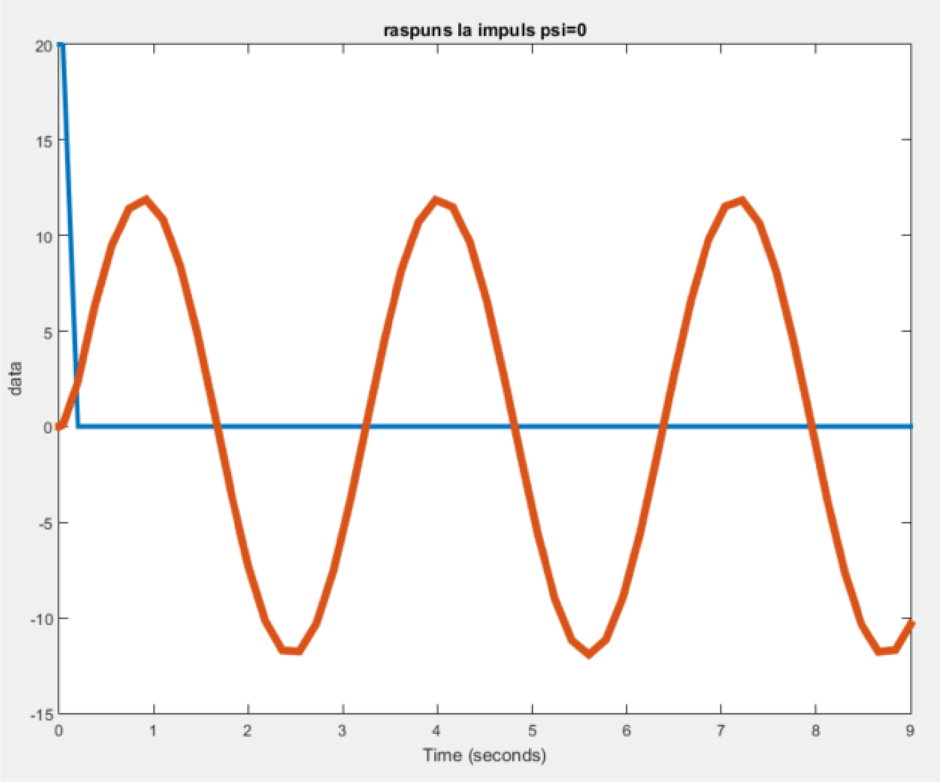
$$H(s) = \frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

K- amplificare;

ξ – factor de amortizare;

ω_n – pulsatia naturala.





Oscilant

ξ =0

ω_n =2

K=1.5

Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin II) partea teoretica

$$H(s) = \frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Oscilant amortizat

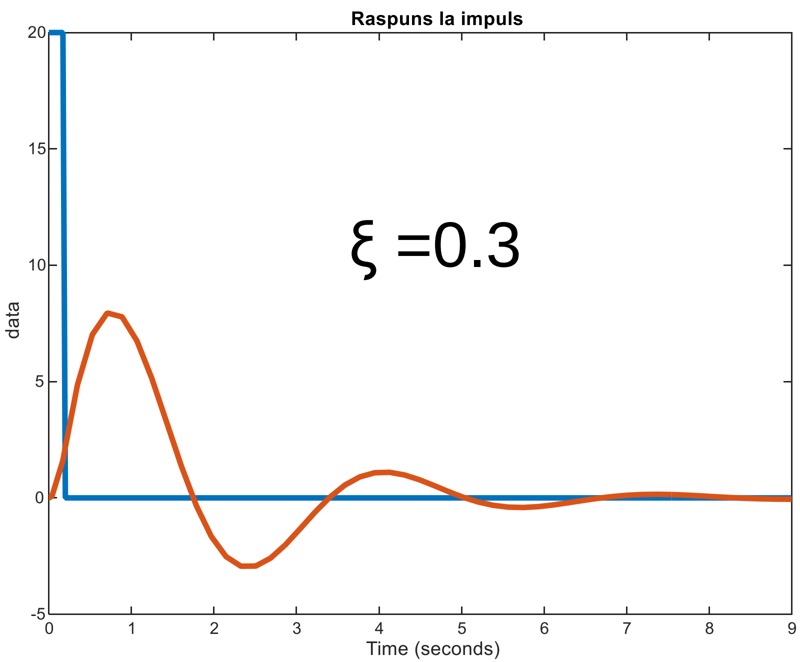
K- amplificare;

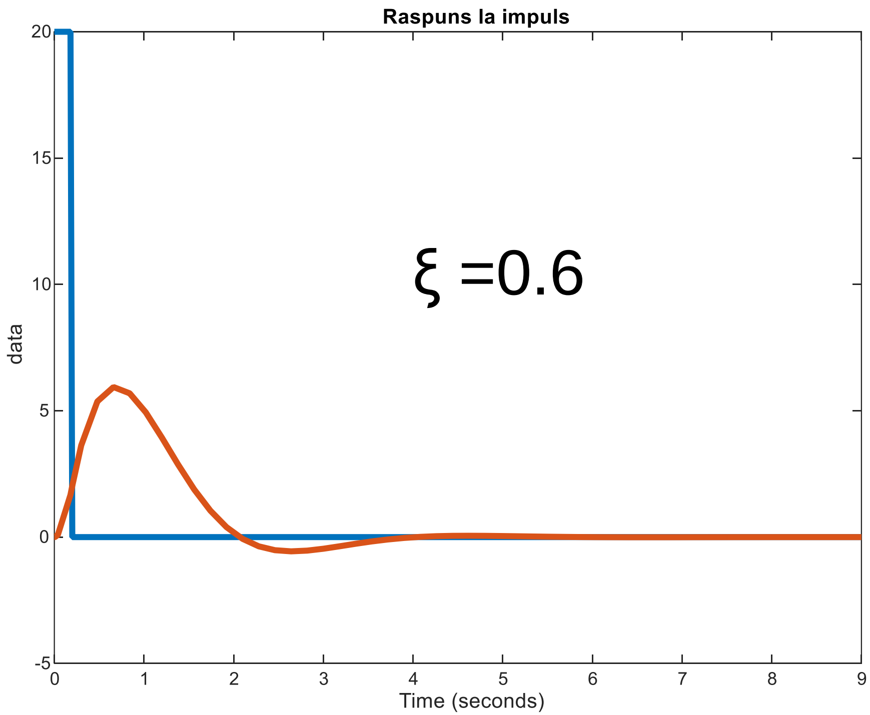
ξ – factor de amortizare;

ω_n – pulsatia naturala.

$\omega_n = 2$

K=1.5





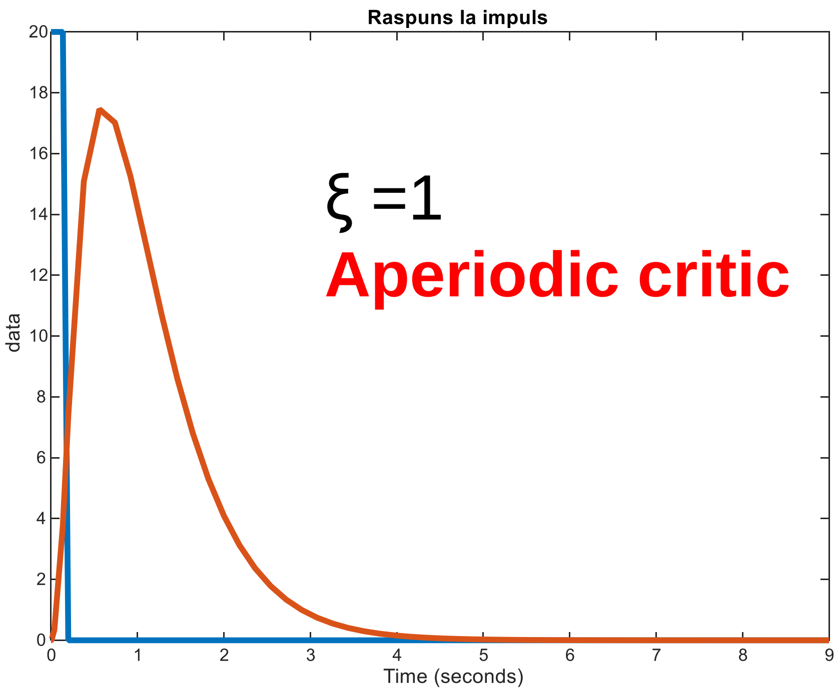
Modele grafice – raspunsul la impuls

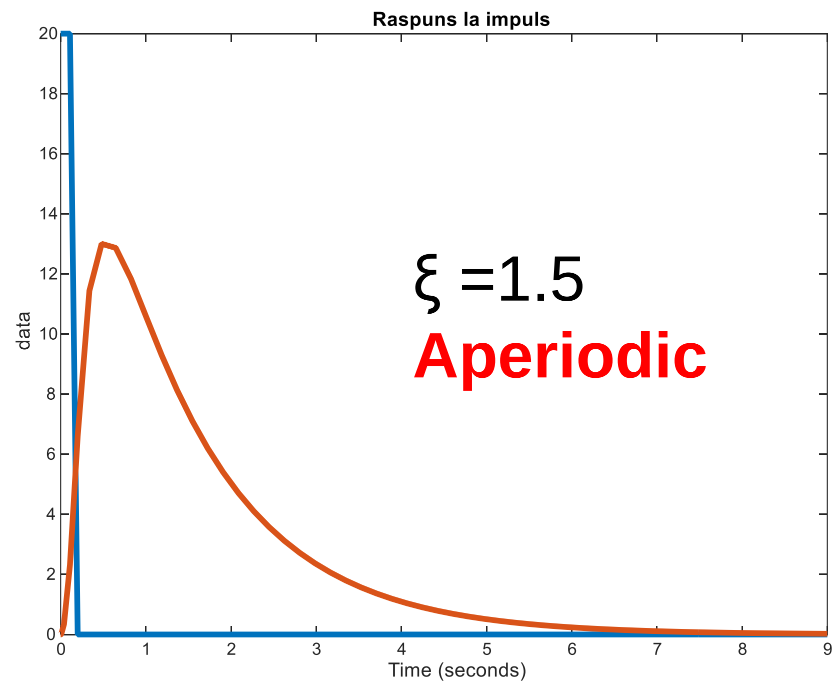
Raspunsul la impuls (sistem de ordin II) partea teoretica

$$H(s) = \frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

K- amplificare;
ξ – factor de amortizare;
ω_n – pulsatia naturala.

$\omega_n = 2$
K=6

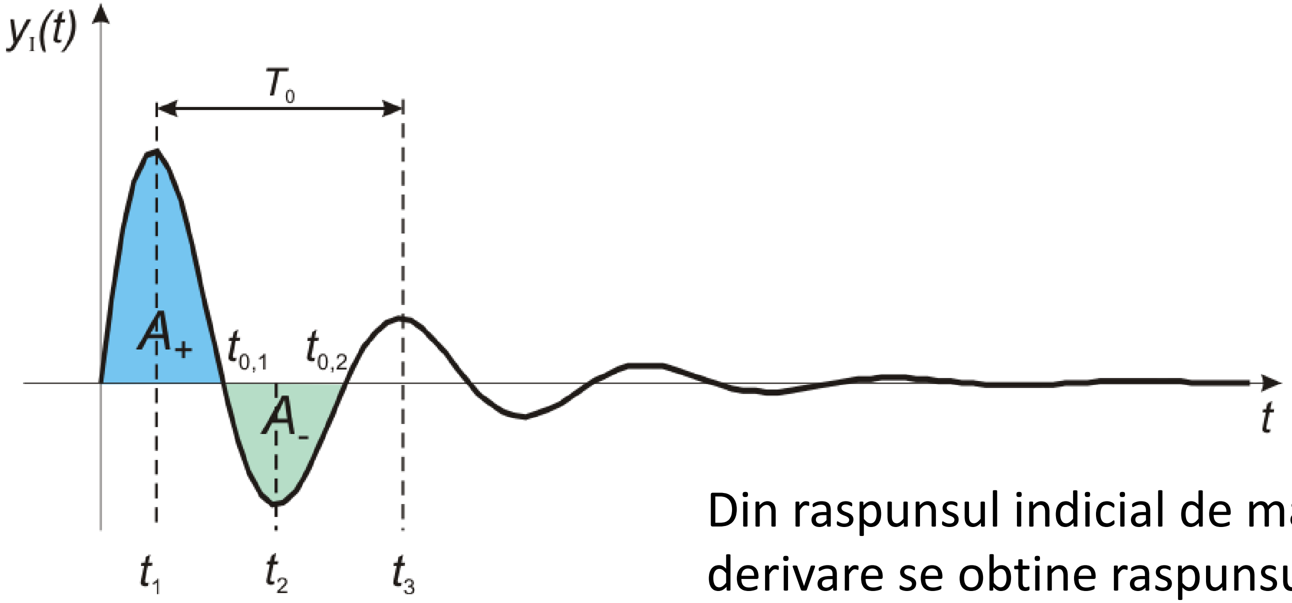




Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin II) partea teoretica

Cazul raspunsului oscilant (de obicei cel mai util)



Din raspunsul indicial de mai jos, prin derivare se obtine raspunsul la impuls $y_I(t)$.

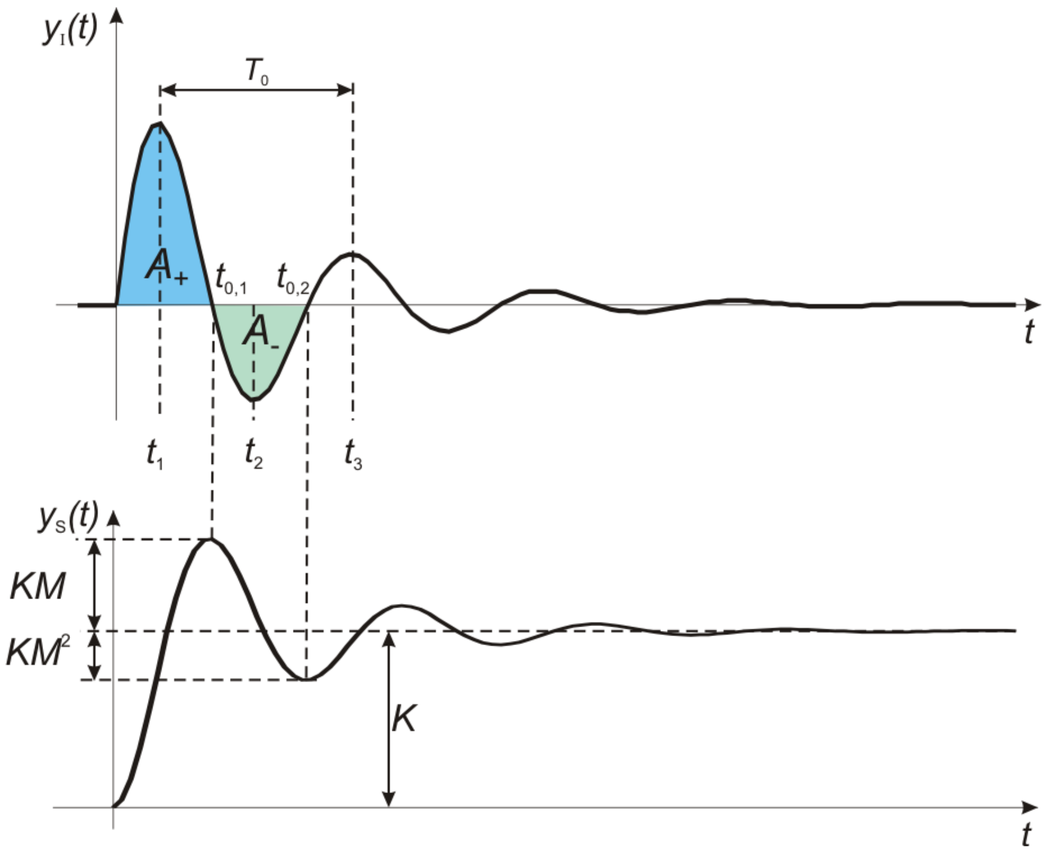
$$y(t) = K\left[1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}\,t + \arccos\xi)\right]$$

$$y_I(t) = \frac{K\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}\,t)$$

Perioada de oscilatie $T_0=t_3-t_1=2(t_2-t_1)$

Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin II) partea teoretica – raspuns oscilant



$$y_S(t) = \int_0^t y_I(\tau)d\tau$$

Tinem cont de valorile la varf si minima din raspunsul indicial $y_S(t)$.

$$A_+ = \int_0^{t_{0,1}} y_I(\tau)d\tau = y_S(t_{0,1}) = K + KM, \qquad A_- = -\int_{t_{0,1}}^{t_{0,2}} y_I(\tau)d\tau =$$

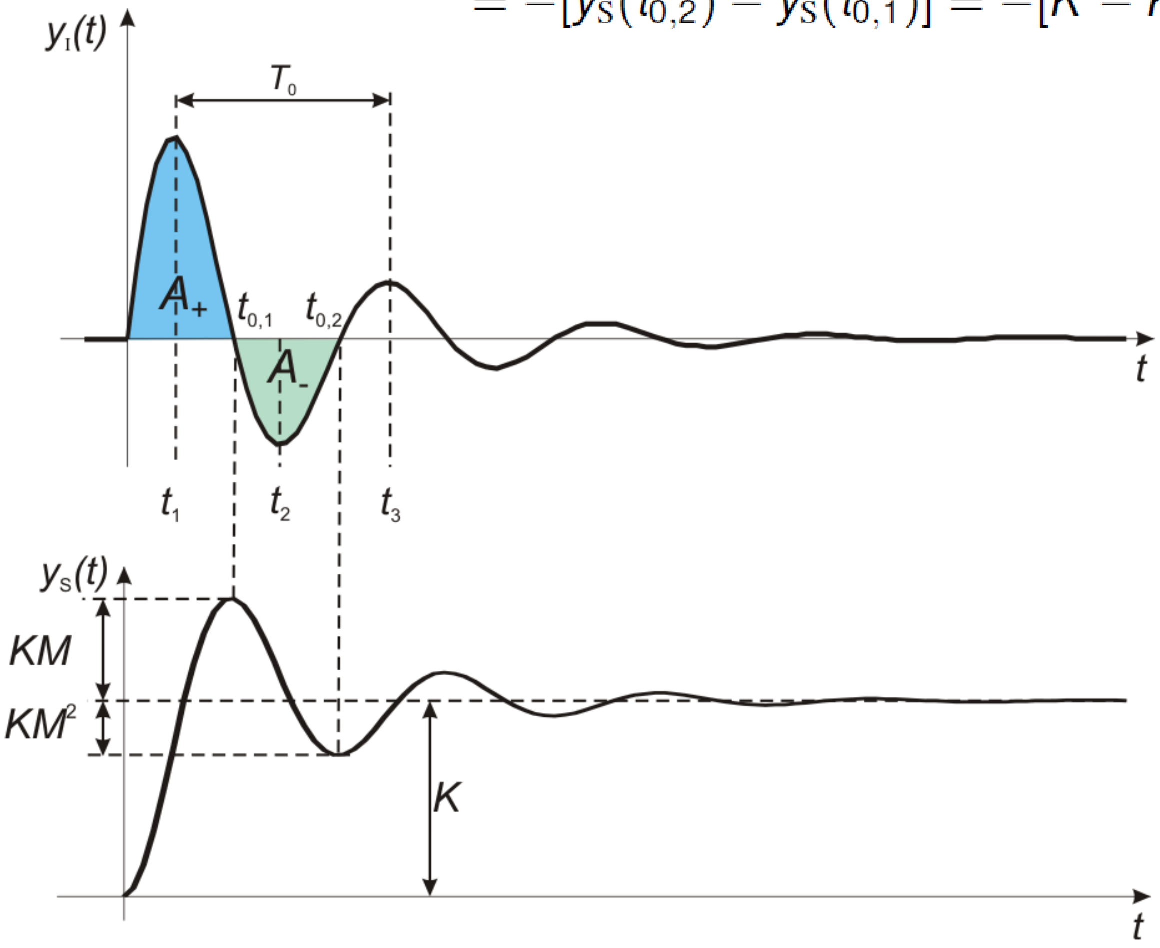
$$= -[y_S(t_{0,2}) - y_S(t_{0,1})] = -[K - KM^2 - (K + KM)] = KM^2 + KM$$

Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin II) partea teoretica – raspuns oscilant

$$A_+ = \int_0^{t_{0,1}} y_I(\tau)d\tau = y_S(t_{0,1}) = K + KM, \qquad A_- = -\int_{t_{0,1}}^{t_{0,2}} y_I(\tau)d\tau =$$

$$= -[y_S(t_{0,2}) - y_S(t_{0,1})] = -[K - KM^2 - (K + KM)] = KM^2 + KM$$



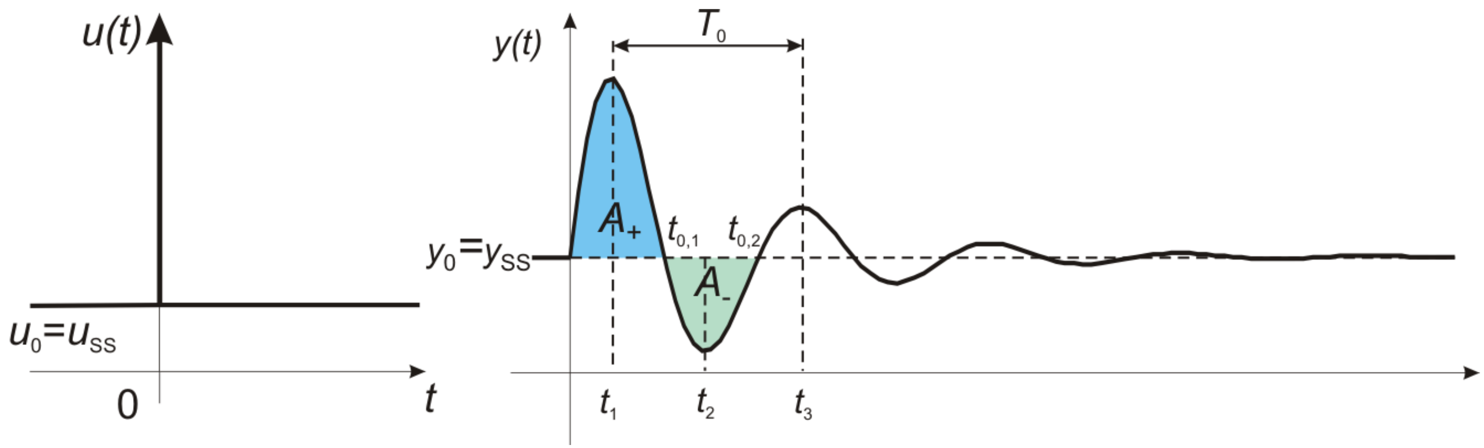
de unde:

$$\frac{A_-}{A_+} = \frac{KM^2 + KM}{K + KM} = M$$

Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin II) partea teoretica – raspuns oscilant

Conditii initiale nenule:



Pentru conditii initiale nenule ($u_0=u_{ss}$; $y_0=y_{ss}$), impulsul este decalat cu u_0: $u(t)=u_0+u_I(t)$, si va conduce la un raspuns decalat $y(t)=y_0+y_I(t)$.

Din valorile de regim stationar se determina amplificarea **$K=y_{ss}/u_{ss}$.**

T_0 nu se modifica, dar trebuie determinate ariile A+ si A_, fata de noile valori de regim stationar.

$$A_+ = \int_0^{t_{0,1}} (y(\tau) - y_0)d\tau \qquad = K + KM$$

$$A_- = -\int_{t_{0,1}}^{t_{0,2}} (y(\tau) - y_0)d\tau = \int_{t_{0,1}}^{t_{0,2}} (y_0 - y(\tau))d\tau \qquad = KM^2 + KM$$

Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin II) partea teoretica – raspuns oscilant

Algoritmul determinarii parametrilor (raspunsul la impuls provine de la un sistem necunoscut)

1. Prin citirea pe grafic a y_{ss}, si stiind din date u_{ss} , se determina amplificarea **$K=y_{ss}/u_{ss}$**.

2. Se citesc pe grafic valorile timpilor $t_{0,1}$ si $t_{0,2}$, pentru care y(t) intersecteaza y_{ss}. Se calculeaza ariile $A_+ = \int_0^{t_{0,1}} (y(\tau) - y_0) d\tau$, $A_- = \int_{t_{0,1}}^{t_{0,2}} \left(y_0 - y(\tau)\right) d\tau$, si se gaseste $\boldsymbol{M = \frac{A_-}{A_+}}$

3. Factorul de amortizare se determina: $\boldsymbol{\xi = \frac{ln\frac{1}{M}}{\sqrt{\pi^2 + ln^2 M}}}$

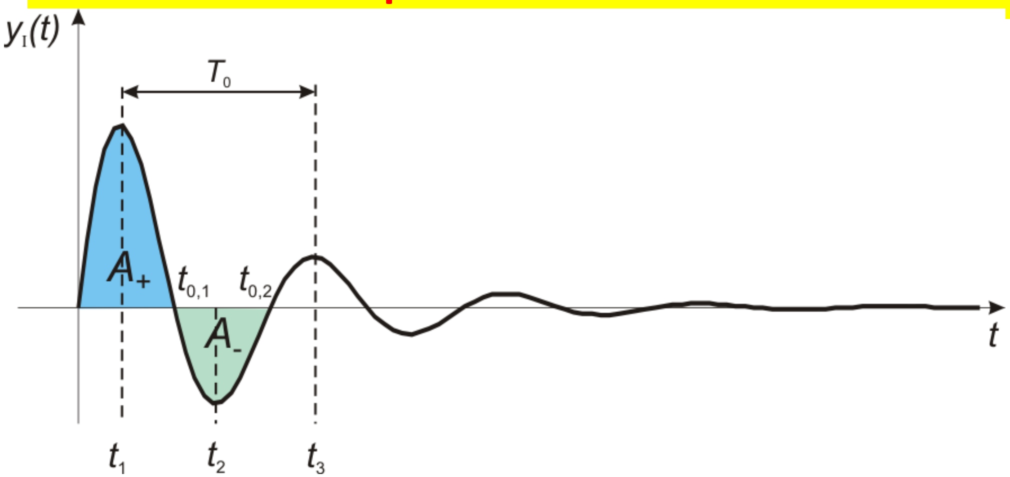
4. Se citesc valorile timpilor t_1, t_2, t_3 si se determina: **$T_0=t_3-t_1$** sau **$T_0=2(t_2-t_1)$**.

5. Si pulsatia naturala: $\boldsymbol{\omega_n = \frac{2\pi}{T_0\sqrt{1-\xi^2}}}$, sau $\boldsymbol{\omega_n = \frac{2}{T_0}\sqrt{\pi^2 + ln^2 M}}$

Modele grafice – raspunsul la impuls

Raspunsul la impuls (sistem de ordin II) partea teoretica – raspuns oscilant

Determinare amplificare in conditii initiale nenule:



Daca se rezolva dy(t)/dt=0 pentru a obtine t_1 la primul varf, si se inlocuieste valoarea lui t_1 in y(t) pentru determinarea valorii raspunsului la varf:

$$y(t_1) = K\omega_n e^{-\frac{\xi \arccos \xi}{\sqrt{1-\xi^2}}}$$

Pentru a estima K:

$$K = \frac{y(t_1)}{\omega_n e^{-\frac{\xi * \arccos(\xi)}{\sqrt{1-\xi^2}}}}$$

ξ si ω_n au fost determinate anterior si nu depind de conditiile initiale.

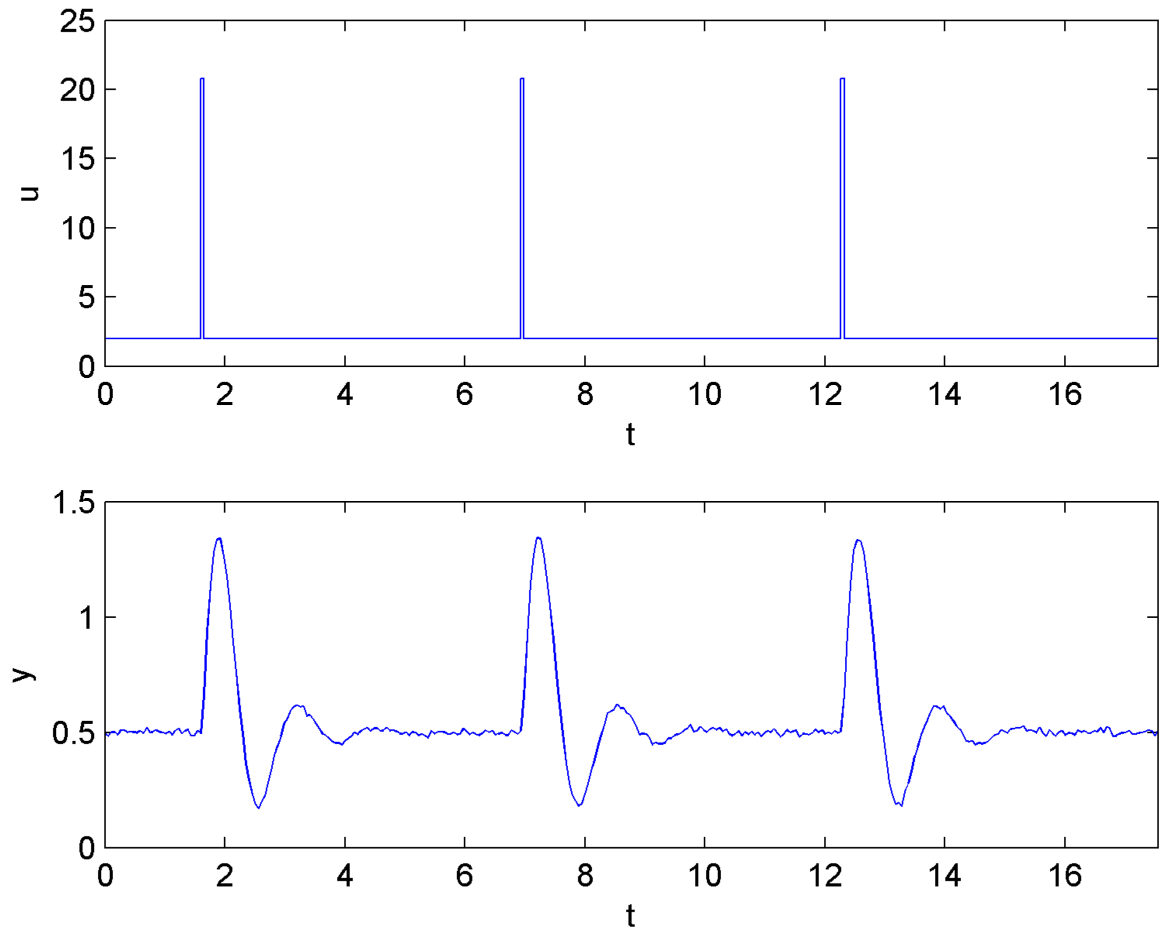
Totusi, ca si in cazul sistemelor de ordin I, modul acesta de calcul este mai putin precis decat modul calculului amplificarii pe baza valorilor de regim stationar.

Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor pentru model de system de ordin II (exp. L Busoniu, System Identification UTCj)

330 esantioane; perioada de esantionare T_e=0.053 s.

Conditii initiale nenule.



Identificare: primul impuls; Validare: impulsurile 2 si 3

Modele grafice – raspunsul la impuls

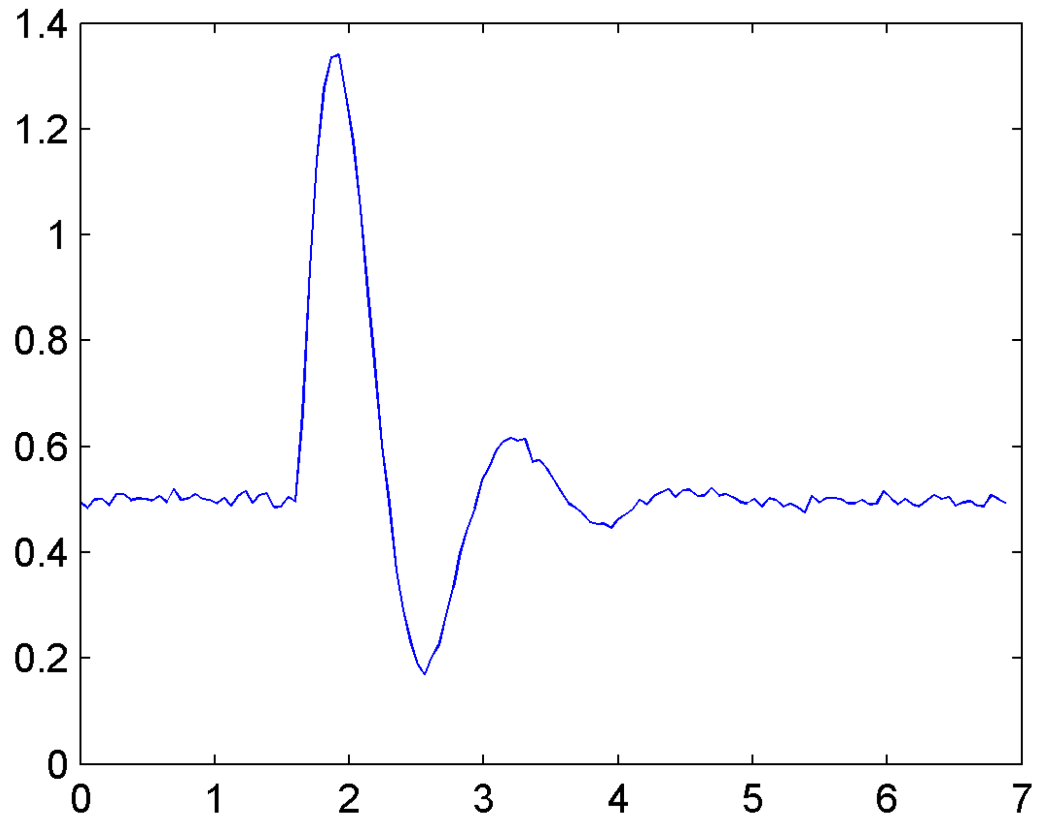
Exemplu :Raspunsul la impuls - determinarea parametrilor pentru model de sistem de ordin II (exp. L Busoniu, System Identification UTCj)

Folosim un model grafic (neparametric) si dorim sa estimam o functie de transfer (model parametric).

$u_0 = u_{ss} = 2$

Pentru iesirea in regim stationar se face media ultimelor valori citite:

$$y_{ss} = y_0 \approx \frac{1}{11} \sum_{k=120}^{130} y(k) \approx 0.5$$

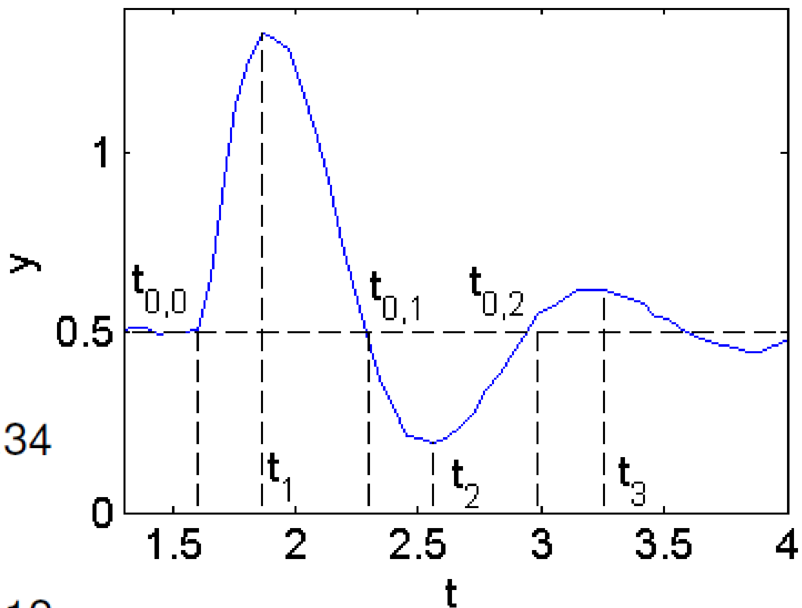


Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor pentru model de sistem de ordin II (exp. L Busoniu, System Identification UTCj)

Estimare factor de amortizare:

- Se citesc: $t_{0,0} \approx 1.6s$ (impuls aplicat in acest moment), $t_{0,1} \approx 2.3s$; $t_{0,3} \approx 2.99s$.



Ariile se estimeaza numeric:

$$A_+ = \int_{t_{0,0}}^{t_{0,1}} (y(\tau) - y_0) d\tau \approx T_s \sum_{k=k_{0,0}}^{k_{0,1}} (y(k) - y_0) \approx 0.34$$

$$A_- = \int_{t_{0,1}}^{t_{0,2}} (y_0 - y(\tau)) d\tau \approx T_s \sum_{k=k_{0,1}}^{k_{0,2}} (y_0 - y(k)) \approx 0.12$$

Unde : $k_{0,0}$; $k_{0,1}$; $k_{0,2}$ sunt indicii corespunzatori timpilor : $t_{0,0}$; $t_{0,1}$; $t_{0,2}$

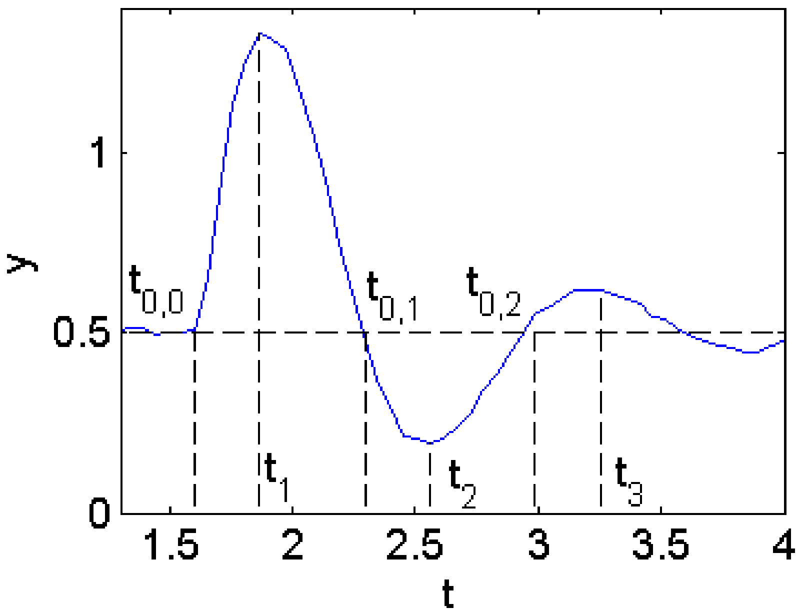
Cu aceste arii calculate se determina:

$$M = \frac{A_-}{A_+} \approx 0.36 \qquad \xi = \frac{\log 1/M}{\sqrt{\pi^2 + \log^2 M}} \approx 0.31$$

Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor pentru model de sistem de ordin II (exp. L Busoniu, System Identification UTCj)

Estimare pulsatie naturala:
- Se citesc pe grafic timpii pentru varfuri: **$t_1 \approx 1.92$s $t_3 \approx 3.2$s**.



Se determina: **$T_0 = t_3 - t_1 = 1.28$s**

Si deci: $$\omega_n = \frac{2\pi}{T_0\sqrt{1-\xi^2}} \approx 5.16$$

Modelul tip functie de transfer, estimat:

$$\widehat{K} = 0.25$$

$$\widehat{\xi} = 0.31$$

$$\widehat{\omega_n} = 5.16$$

$$\widehat{H}(s) = \frac{\widehat{K}\widehat{\omega}_n^2}{s^2 + 2\widehat{\xi}\widehat{\omega}_n s + \widehat{\omega}_n^2} = \frac{6.64}{s^2 + 3.21s + 26.68}$$

Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor – sistem de ordin II (exp. L Busoniu, System Identification UTCj)

Pentru o simulare care ia in calcul si conditiile initiale:

- Trecem la modelul in spatiul starilor:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

- De la functia de transfer se trece in domeniul timp:

$$\ddot{y}(t) = -2\xi\omega_n\dot{y}(t) - \omega_n^2 y(t) + K\omega_n^2 u(t)$$

- Se adopta variabile de stare $x_1=y$ si $x_2=dy/dt$ (sistem de ordin II), identificandu-se matricile A, B, C, D:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}x_2(t)\\ -2\xi\omega_n x_2(t) - \omega_n^2 x_1(t) + K\omega_n^2 u(t)\end{bmatrix}$$
$$= \begin{bmatrix}0 & 1\\ -\omega_n^2 & -2\xi\omega_n\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ K\omega_n^2\end{bmatrix}u(t)$$
$$y(t) = x_1(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + 0u(t)$$

Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor – sistem de ordin II (exp. L Busoniu, System Identification UTCj)

Pentru o simulare care ia in calcul si conditiile initiale:

- Rezulta:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -26.68 & -3.22 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 6.64 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + 0u(t)$$

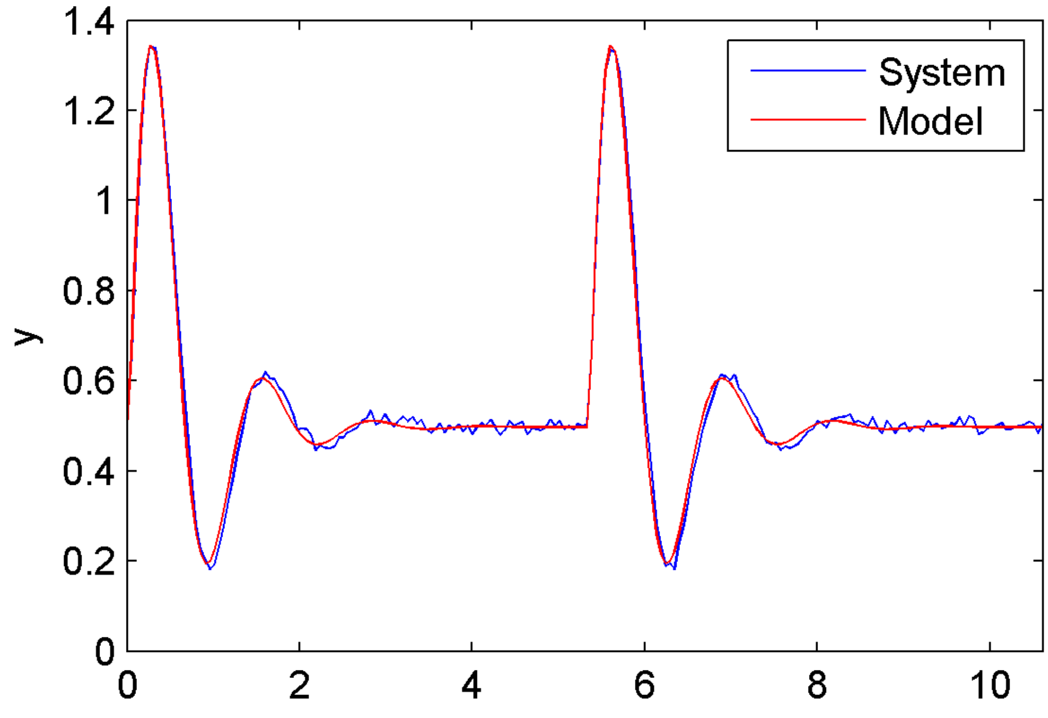
- Cu vectorul de stare : $\mathrm{X}(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}$

Modele grafice – raspunsul la impuls

Exemplu :Raspunsul la impuls - determinarea parametrilor – sistem de ordin II (exp. L Busoniu, System Identification UTCj)

VALIDARE model:

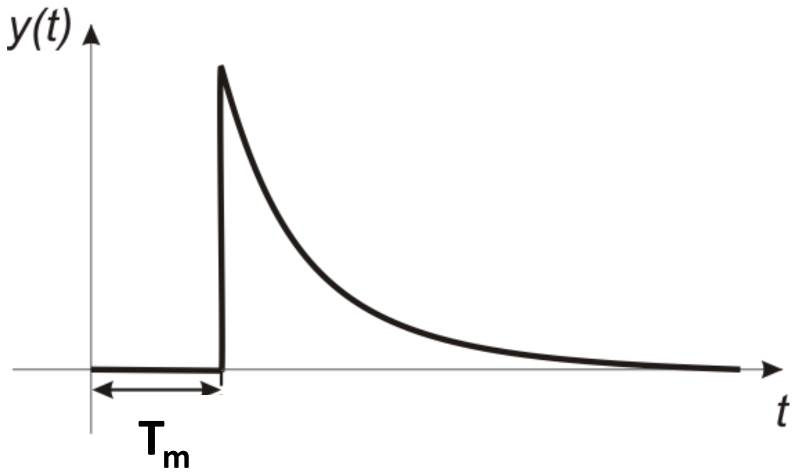
La simulare, se iau in considerare valorile initiale: $X_1(0)=y_0$; $X_2(0)=0$ (pornire din regim stabilizat dy(0)/dt=0)

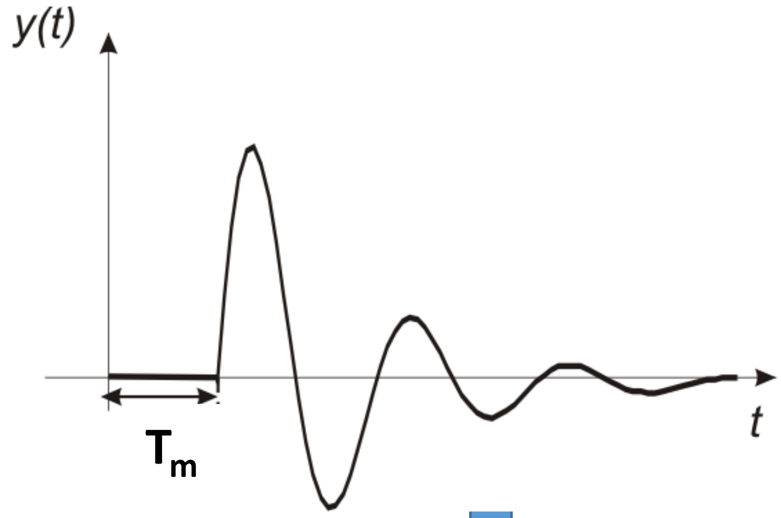


Suma erorilor patratice pe datele de validare:

$$J = \frac{1}{N}\sum_{k=1}^{N} e^2(k) \approx 8 \cdot 10^{-4}$$

Sisteme cu timp mort (T_m)





$$H(s) = \frac{K}{sT + 1} e^{-sT_m}$$

$$\mathrm{H(s)} = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} e^{-sT_m}$$

Si raspunsul la impuls poate fi unul cu intarziere.

De pe grafic se citeste timpul mort T_m. Se scriu functiile de transfer, ca mai sus.